# An OFDM-Based Multipath Channel Sounding Method with Fractional Delays Resolution

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## Introduction

Conventional orthogonal frequency-division multiplexing (OFDM) systems often use frequency-domain channel estimation techniques [1,2], which assume the composite channel impulse response (CIR) to be sampled at the receiver sampling period. Hence, the physical multipath delays can only be resolved when they occur at integer samples. Such a result is not satisfactory, however, for the high-resolution sounding of the physical multipath fading channel, since it is rather rare for the delays to occur right at integer samples. In this paper, an (OFDM)-based high resolution multipath fractional delay channel sounding (CS) system is proposed. Aware of the known transmit and receive filters and the sinusoidal combination feature of channel frequency response (CFR), we apply an iterative maximum likelihood (ML) procedure for sinusoidal parameter estimation, which is a simplification of the alternating notch-periodogram algorithm (ANPA) [3] in frequency domain to catch the channel path delays. Besides, the proposed CS method is an aliasing-free technique since it is operated under *T/2*-spaced sampling. According to simulation results, the ITU Vehicular A channel model can be sounded by the proposed system with high precision in fractional delays, and the mean squared error (MSE) performance of the reconstructed CFR is greatly improved over the conventional least square (LS) CFR estimate.

## Problem Formulation

In the OFDM channel sounding system, the $i^{th}$ OFDM sounding symbol (an $N$-point unit-modulus preamble sequence) $P_{i,m}$ are modulated onto $N$ subcarriers by using the $N$-point inverse Fast Fourier transform (IFFT) at transmitter. The TX sampling clock is denoted as $T$. After adding a guard interval $T_g$ , the transmit pulse shaping is applied by using a square-root raised cosine filter (SRRC) with roll-off factor $0<\beta<1$. Then the OFDM signal is transmitted over a multipath fading channel which has an unknown number of paths $L$ and associated complex attenuations $\{h_l\}$ and delays $\{\tau_l\}$ for $l$=1,…, $L$-1. At the receiver, the same SRRC filter as the TX filter is applied, and the received signal is sampled at $T/2$-spaced clock for preventing spectral aliasing. Assuming the guard interval is longer than the maximum channel excess delay and the synchronization is perfect, the guard interval is correctly removed, and then the left-right shifted version of the 2$N$-point FFT of the $i^{th}$ OFDM sounding symbol block can be represented as

$$\mathbf{R}_i = \mathbf{P}_i \odot \mathbf{Z} \odot \mathbf{H} + \mathbf{V}, \tag{1}$$

where $\odot$ denotes pairwise multiplication of two vectors, $\mathbf{P}_i$ is the preamble symbol vector with padding zeros at both sides, $\mathbf{Z}$ is the known raised cosine (RC) filter's frequency response (FR), $\mathbf{H}$ is the channel frequency response (CFR), and $\mathbf{V}$ is the frequency domain (FD) AWGN noise vector. Note that $\mathbf{H}$ can be expressed as the combination of $L$ different sinusoids:

$$H_k = \sum_{l=1}^{L} h_l e^{-\frac{j2\pi k\tau_l}{NT}}, \ for \ \ -N \le k \le N-1. \tag{2}$$

Since $\mathbf{Z}$ is raised cosine, the useful FD data region $I$ is restricted to $-\left\lceil \left(\frac{1+\beta}{2}\right)N \right\rceil \le k \le \left\lceil \left(\frac{1+\beta}{2}\right)N \right\rceil -1$ .

In the interval $I$, we can first perform the least square (LS) CFR estimation, resulting in

$$\hat{\mathbf{H}}_i = [\mathbf{P}_i \odot \mathbf{Z}]^{-1} \odot \mathbf{R}_i. \tag{4}$$

However, the above LS CFR estimation would cause noise enhancement problem in the RC filter's roll-off regions. The noise component in (4) can be presented as

$$W_{i,k} = \eta_k V_{i,k},\ -\left\lceil \left(\frac{1+\beta}{2}\right) N \right\rceil \le k \le \left\lceil \left(\frac{1+\beta}{2}\right) N \right\rceil -1, \tag{5}$$

where $\eta_k = 2/Z_k$ is the noise enhanced factor. The above noise enhancement caused by RC filter FR has to be taken into consideration while we are designing the proposed CS method. Hence, it is not recommended to use the full band $I$, and in the next section, the acquired band ratio (ABR) problem will be considered for the proposed CS method.

## The Proposed Channel Sounding Method

In our proposed CS method, the Chu sequence [4] is used as the preambles due to its constant envelope feature. Therefore, we only have to consider the noise enhancement problem caused by the RC filter's FR in (4). Besides, the sinusoidal parameter estimating technique is employed to estimate the path delay element via its iterative operations. In the beginning of the proposed CS method, the received sounding symbols are averaged every $q$ times for reducing the noise effect, where the choice of $q$ depends on the channel coherence time [5]. And the averaged CFR is

$$\tilde{\mathbf{H}} = \frac{1}{q} \sum_{i=0}^{q-1} \hat{\mathbf{H}}_i. \tag{6}$$

Then a FD window $\mathbf{\Pi_1}$ with ABR factor $\rho_I = (1+\beta)/2$ is applied to acquire the CFR in the interval $I$. Here, the averaged and windowed LS estimated CFR is modeled as

$$\begin{aligned} \bar{\mathbf{H}}_1 &= \mathbf{\Pi}_1 \odot \tilde{\mathbf{H}} \\ &= \bar{\mathbf{E}}_1(\boldsymbol{\tau})\mathbf{h} + \mathbf{\Pi}_1 \odot \mathbf{W}, \end{aligned} \tag{7}$$

where

$\Pi_{1,k} = 1,\ for\ k \in I,$ and $\Pi_{1,k} = 0,$ elsewhere

$\bar{\mathbf{e}}_1(\tau_l) = \mathbf{\Pi}_1 \odot [e^{-j2\pi\frac{\tau_l(-N)}{NT}}, \cdots, e^{-j2\pi\frac{\tau_l(N-1)}{NT}}]^T$ is called the windowed signal vector

$\bar{\mathbf{E}}_\mathbf{1}(\boldsymbol{\tau}) = [\bar{\mathbf{e}}_1(\tau_0)\ \bar{\mathbf{e}}_1(\tau_1) \cdots \bar{\mathbf{e}}_1(\tau_{L-1})]$ is a $2N \times L$ windowed Vandermonde matrix

Then the CFR $\bar{\mathbf{H}}_1$ in (7) is taken by the proposed $T/2$-spaced high resolution CS method for acquiring the channel order, path fractional delays, and path attenuations iteratively. The CS method is simplified from the ANPA algorithm [3]. As show in Fig. 1, the procedure consists of three major iteration steps: (A) CFR notching using the previously estimated path delays, (B) IFFT-based searching for new path delay, and (C) path order testing based on the EDC criterion. Let $m$ be the iteration index. Initially, we set $m$=0 and $\hat{\boldsymbol{\tau}}_0 = [\ ]$, which is the initial notch set. We then describe the iterative operations below.

In step (A), the $m^{th}$ notched CFR is defined as

$$\mathbf{\Phi}_m = \begin{cases} \bar{\mathbf{H}}_1,\ for\ m = 0 \\ \mathbf{P}^\perp(\hat{\boldsymbol{\tau}}_m)\bar{\mathbf{H}}_1,\ for\ m \ge 1 \end{cases}, \tag{8}$$

where $\hat{\boldsymbol{\tau}}_m = [\hat{\tau}_1\ \hat{\tau}_2 \cdots \hat{\tau}_m]$ is the set of the previously estimated delays, $\mathbf{P}^\perp(\hat{\boldsymbol{\tau}}_m)$ is the orthogonal projection matrix for notching the estimated paths as given by

$$\mathbf{P}^\perp(\hat{\boldsymbol{\tau}}_m) = \mathbf{I} - \bar{\mathbf{E}}_1(\hat{\boldsymbol{\tau}}_m)^H [\bar{\mathbf{E}}_2(\hat{\boldsymbol{\tau}}_m)^H \bar{\mathbf{E}}_2(\hat{\boldsymbol{\tau}}_m)]^{-1} \bar{\mathbf{E}}_2(\hat{\boldsymbol{\tau}}_m)^H, \tag{9}$$

$$\bar{\mathbf{E}}_2(\hat{\boldsymbol{\tau}}_m) = diag(\mathbf{\Pi}_2)\bar{\mathbf{E}}_1(\hat{\boldsymbol{\tau}}_m), \tag{10}$$

and similar to $\mathbf{\Pi}_1$, $\mathbf{\Pi}_2$ is the second rectangular window with ABR factor $\rho_2$=0.5 for acquiring the 3dB bandwidth for path cancellation. Note that in (8)-(9), $[\bar{\mathbf{E}}_2(\hat{\boldsymbol{\tau}}_m)^H \bar{\mathbf{E}}_2(\hat{\boldsymbol{\tau}}_m)]^{-1} \bar{\mathbf{E}}_2(\hat{\boldsymbol{\tau}}_m)^H \bar{\mathbf{H}}_1$ serves as the channel attenuation estimate for the current delay set $\hat{\boldsymbol{\tau}}_m$. In step (B), the weighting vector $\breve{\eta}_k = \eta_k^{-1}$ is first applied to $\mathbf{\Phi}_m$ for decreasing both the noise effect and sinc-like side lobes caused by rectangular window. Then the IFFT is performed to catch the CIR, and the CIR's peak is found by zoom-IFFT for a more accurate fractional delay estimate $\hat{\tau}_{m+1}$ associated with a new path. In (C), the efficient detection criterion (EDC) [6] is calculated to determine the path order :

$$EDC(m) = \Upsilon \log \|\mathbf{\Phi}_m\|^2 + \frac{3m}{2}\alpha \log \Upsilon, \; for \; m \geq 0, \; \alpha > 0, \tag{11}$$

where $\Upsilon = \lceil (1-\beta)N \rceil$ is the length of non-noise enhancement part, and $\alpha$ is the EDC penalty factor. By finding the minimizing index of the EDC(m) criterion, or simply by observing the index at which *EDC*(*m*) > *EDC*(*m-1*), the estimated path order is given by

$$\hat{L} = \arg\min_m EDC(m) \,. \tag{12}$$

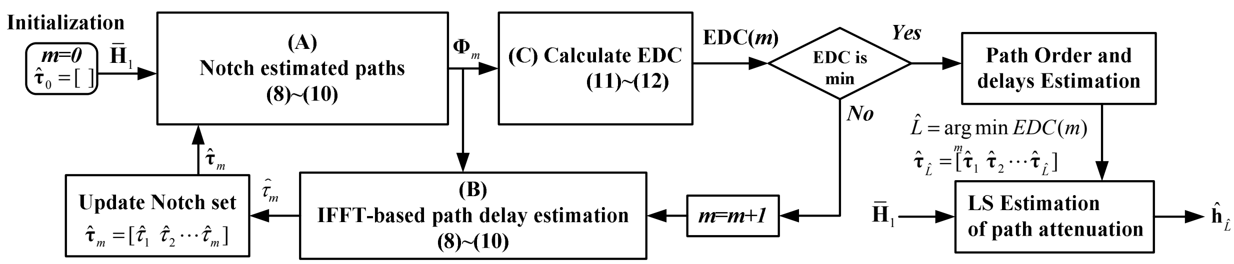


Fig. 1 The proposed CS method

After finding the path order $\hat{L}$ and the set $\hat{\boldsymbol{\tau}}_{\hat{L}}$ of path delays estimate, the path attenuations $\hat{\mathbf{h}}_{\hat{L}}$ can be estimated by the linear LS method from $\bar{\mathbf{H}}_1$.

## Simulation Results

In our simulation, the number of subcarriers $N$ is 256, signal bandwidth is 20MHz, sampling interval ($T$) is 0.05$\mu$sec, guard interval ($T_g$) is 6.4$\mu$sec. Besides, the RC roll-off factor ($\beta$) is 0.5, symbol averaging number ($q$) is 10, penalty factor $\alpha$ is [0.5 2], carrier frequency ($f_c$) is 2.4 GHz, and Doppler frequency ($f_d$) is 100Hz (45km/hr). The MSE of reconstructed CFR is used to evaluate the CS system's overall performance. Note that the theoretical CFR-MSE performance is calculated in terms of the LS estimation of path attenuations under known path order and delays.

As the first example, we simulate the extreme case of two paths. In Fig. 2, different delays and path attenuations are used for testing the proposed CS method's performance. It is observed that the delay estimation result would degraded while the path delays become closer, since the strong interpath interference (IPI) would affect the accuracy of path detection and delay estimation. However, the proposed method is able to resolve the fractional spacing between the two paths, and significantly outperforms the conventional LS method in terms of the reconstructed CFR. In Fig. 3, we show the CFR-MSE performance for the ITU Vehicular A channel model, which is specified by the normalized delays [0 6.2 14 21.8 34.6 50.2]*$T$, and averaged powers [0 -1 -9 -10 -15 -20] (dB). It is observed that the EDC penalty factor $\alpha$ should be tuned smaller under the lower SNR environment for increasing the path detecting probability. However, regarding to the CFR-MSE performance, the different setting of $\alpha$ would only affect the result slightly. This indicates that the proposed method is more robust and accurate for multipath channel sounding and estimation, as compared to the conventional LS estimate. The simulation results also verify that the proposed CS method can accurately acquire the physical multipath fading channel model with fractional delays. It has a CFR-MSE performance close to the theoretical value.

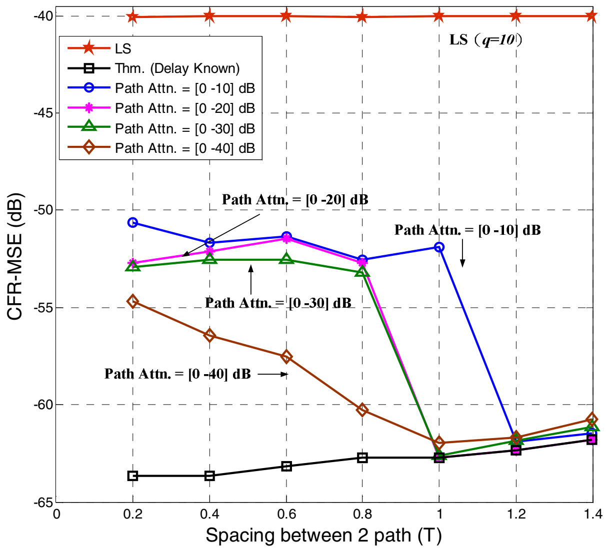


Fig. 2 The CFR-MSE performance vs. the delay spacing of the two-path channel (SNR=30dB, $\alpha$=2)

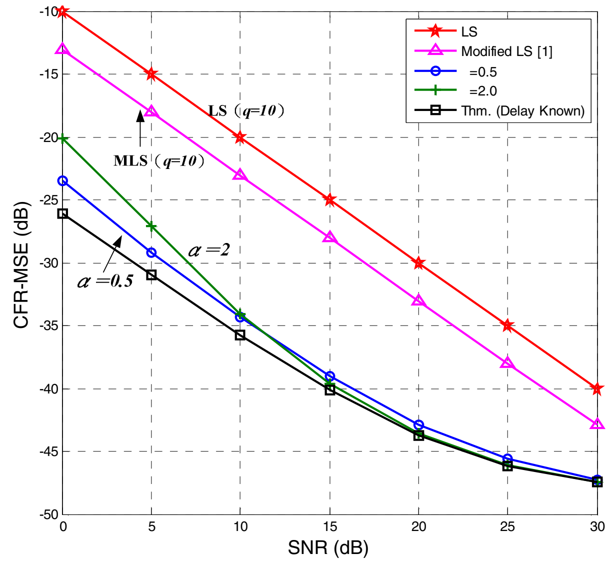


Fig. 3 The CFR-MSE Performance vs. SNR for ITU Vehicular A channel model

## Conclusions

In this paper, an OFDM-based high resolution CS method is proposed under *T/2*-spaced sampling. The proposed CS method consists of three iterative steps. In each iteration, it attempts to catch a new path with accurate fractional delay. Besides, it is equipped with EDC criterion for path order determination. The CFR-MSE performance can be greatly improved over the conventional LS method. Besides, the proposed CS method is robust enough for different settings of EDC penalty factor $\alpha$. In addition, some algorithmic refinements could be added for decreasing the complexity in the future.

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