CMAC-Based Fault Accommodation Control for Tank System

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Abstract - This paper presents a learning approach using cerebellar model articulation controller (CMAC) to accommodate faults for a class of multivariable nonlinear systems. A CMAC is proposed to estimate the unknown fault. Then, an adaptive fault accommodation controller is derived based on Lyapunov function, so that the proposed control system can accommodate the faults with desired system stability. Finally, the proposed fault accommodation control system is applied to a tank control system. Simulation results show that the proposed method can effectively achieve the fault accommodation for this system.

Keywords: cerebellar model articulation controller (CMAC), fault accommodation, tank system.

I. INTRODUCTION

In the presence of a failure, an on-line approximator can be used to estimate the possible fault function. The fault accommodation requires that the fault is self-corrected, usually through control reconfiguration [1, 2]. Recently neural networks have been applied for system identification and control [3, 4]. The most useful property of neural networks is their ability to uniformly approximate arbitrary input-output mappings on closed subsets. Several fault accommodation controls using neural networks have been proposed [5, 6]. However, the learning of neural network is slow since all the weights are updated during each learning cycle. Therefore, the effectiveness of the neural network is limited in problems requiring on-line learning.

Cerebellar model articulation controller (CMAC) is classified as a non-fully connected perceptron-like associative memory network with overlapping receptive-fields [7]. It has been applied to identify the nonlinear functions for its good generalization capability and fast learning property. The CMAC has been already validated to be able to approximate a nonlinear function over a domain of interest to any desired accuracy; and it intends to solve the fast size-growing problem and the learning difficult in currently available types of neural networks [8-10]. However, the conventional CMAC uses local constant binary receptive-field basis function. The disadvantage is that its output is constant within each quantized state and the derivative information is not preserved. For acquiring the derivative information of input and output variables, Chiang and Lin developed a CMAC network with differentiable Gaussian receptive-field basis function, and provided the convergence analyses of this network [11]. This makes CMAC a suitable candidate for a wide class of unknown system identification. This paper presents a fault accommodation design method for a class of multivariable nonlinear systems by using a CMAC to estimate the unknown fault. Finally, a tank control system is simulated to illustrate the effectiveness of the proposed design method.

II. PROBLEM FORMULATION

Consider a class of multivariable nonlinear dynamic systems described by [1], [6]

$$\dot{x}(t) = F(x(t),t)$$
$$+ G(x(t),t)[u(t) + \eta(x(t),t) + B(t-T)f(x(t),t)] \quad (1)$$

where $x \in R^m$ is the state vector, $u \in R^m$ is the input, $B(t-T) \in R^{m \times m}$ is a matrix function

representing the time profile of failures where $T \in R^+$ is unknown time of occurrence of faults. The vector $F \in R^n$ and matrix $G \in R^{n \times m}$ represent the nominal model dynamics. The unknown nonlinear function $f \in R^m$ characterizes the change in the system due to a failure. The nonlinear function $\eta \in R^m$ represents system uncertainty including modeling error and disturbance, each component η_i of the system uncertainty η are assumed to be unstructured and bounded in a domain of interest:

$$|\eta_i(x,t)| \le \bar{\eta}_i, \ i = 1,2,\cdots m \quad (2)$$

where $\bar{\eta}_i, \ i = 1,2,\cdots m$ are positive constants. The diagonal B represents the time profiles of the fault

$$B(t-T) \equiv diag(\beta_1(t-T)\cdots\beta_m(t-T)) \quad (3)$$

III. FAULT ACCOMMODATION USING CMAC NETWORK

Suppose a known nominal controller u_N causes the nominal system to exhibit some desired behavior. Consider a corrective control function to be designed, such that the control law is assumed to take the following form:

$$u = u_N + u_C \quad (4)$$

where u_C is the corrective control with fault accommodation.

Since CMAC has been validated to be able to approximate a nonlinear function with desired accuracy, it is utilized as the corrective controller to achieve fault accommodation.

A. CMAC Network

The architecture of CMAC is depicted in Fig. 1. The signal propagation and the basic function in each space of CMAC are introduced as follows.

1) Input space S : For a given n-dimensional input space $S = [s_1, s_2, \cdots, s_n]^T \in R^n$, each input state variable s_j must be quantized into discrete regions (called an *element*) according to given control space. The number of elements, n_E, is termed as a resolution.

2) Association memory space A: Several elements can be accumulated as a *block*. A

denotes an association memory space with n_A ($n_A = n \times n_B$) components. In this space, each block performs a *receptive-field basis function.* The Gaussian function is adopted here as the receptive-field basis function which can be represented as

$$\phi_{jk}(s_j) = \exp\left[\sum_{j=1}^n \frac{-(s_j - m_{jk})^2}{\sigma_{jk}^2}\right], \text{for} \ k = 1,2,\cdots n_B \quad (5)$$

where $\phi_{jk}(s_j)$ presents the kth block of the jth input s_j with the mean m_{jk} and variance σ_{jk}.

3) Receptive-field space R: Areas formed by blocks are called *receptive-fields*. The number of receptive-field, n_R, is equal to n_B in this study. Each location of A corresponds to a receptive-field. The multi-dimensional receptive-field function is defined as

$$\gamma_k(s) = \prod_{j=1}^n \phi_{jk}(s_j) = \exp\left[\sum_{j=1}^n \frac{-(s_j - m_{jk})^2}{\sigma_{jk}^2}\right],$$

$$\text{for} \ k = 1,2,\ldots,n_R \quad (6)$$

where γ_k is associated with the kth receptive-field. The multi-dimensional receptive-field function can be expressed in vector notation as

$$\Gamma = [\gamma_1, \gamma_2, \cdots, \gamma_{n_R}]^T \quad (7)$$

4) Weight memory space W: Each location of R to a particular adjustable value in the weight memory space with n_R components can be expressed as

$$W = [w_1, \cdots, w_q, \cdots, w_m] = \begin{bmatrix} w_{11} \cdots w_{1q} \cdots w_{1m} \\ \vdots \ddots \vdots \ \vdots \ \vdots \\ w_{k1} \cdots w_{kq} \cdots w_{km} \\ \vdots \ \vdots \ \vdots \ddots \vdots \\ w_{n_R1} \cdots w_{n_Rq} \cdots w_{n_Rm} \end{bmatrix} \quad (8)$$

where $w_q = [w_{1q}, \cdots, w_{kq}, \cdots, w_{n_Rq}]^T \in R^{n_R}$, and w_{kq} denotes the connecting weight value of the qth output associated with the kth receptive-field.

5) Output space $\hat{f}$: The output of CMAC is expressed as

$$\hat{f}_q = w_q^T \Gamma = \sum_{k=1}^{n_R} w_{kq}\gamma_k, \text{for} \ q = 1,2,\cdots m \quad (9)$$

It can be expressed in a vector notation as

$$\hat{f}(x,W) = [\hat{f}_1,\ldots,\hat{f}_q,\ldots,\hat{f}_m]^T = W^T \Gamma \quad (10)$$

B. Fault Accommodation Analysis

In this section, a design methodology is illustrated for fault accommodation in a class of nonlinear systems shown in (1). Assumes there exists a bound such that

$$\|\boldsymbol{\eta}(x,t) + \boldsymbol{J}(x,t)\|_{\infty} \le \tau^* \qquad (11)$$

where $\boldsymbol{J}(x,t) = \boldsymbol{f}(x,t) - \hat{f}(x,\boldsymbol{W}^*)$ is the approximation error for a failure function, $\boldsymbol{W}^*$ is an unknown constant matrix that represents the optimal network weight of $\boldsymbol{W}^*$, and τ^* is a constant representing a uncertainty bound. Since the bound τ^* is general unknown, an estimation of this bound denoted by $\hat{\tau}$ will be derived. Assume a Lyapunov function $V_N(x)$ satisfies the following:

$$\alpha_1(\|x\|) \le V_N \le \alpha_2(\|x\|) \qquad (12)$$

$$\frac{\partial V_N}{\partial x}[\boldsymbol{F}(x) + \boldsymbol{G}(x)\boldsymbol{u}_N] \le -\alpha(\|x\|) \qquad (13)$$

where α, α_1 and α_2 are finite positive constants. Consider the following Lyapunov function candidate:

$$V(x,\widetilde{\boldsymbol{W}},\widetilde{\tau}) = V_N(x) + \frac{1}{2\xi_1}t_r(\widetilde{\boldsymbol{W}}^T\widetilde{\boldsymbol{W}}) + \frac{1}{2\xi_2}\widetilde{\tau}^2 \qquad (14)$$

where $\widetilde{\boldsymbol{W}}(t) \equiv \hat{\boldsymbol{W}}(t) - \boldsymbol{W}^*$ is the estimation error of the weight, $\widetilde{\tau} = \hat{\tau} - \tau^*$ is the bound estimation error, and ξ_1 and ξ_2 are the positive constants which present the adaptive gains in the adaptation of $\hat{w}$ and $\hat{\tau}$. From (14), the time derivative of V satisfies that

$$\dot{V} = \frac{\partial V_N}{\partial x}[\boldsymbol{F}(x) + \boldsymbol{G}(x)(\boldsymbol{u} + \boldsymbol{\eta}(x) + \boldsymbol{f}(x))]$$
$$+ \frac{1}{\xi_1}t_r(\widetilde{\boldsymbol{W}}^T\dot{\widetilde{\boldsymbol{W}}}) + \frac{1}{\xi_2}\widetilde{\tau}\dot{\hat{\tau}} \qquad (15)$$

Define the m-dimensional vector

$$\boldsymbol{\psi}(x) \equiv \frac{\partial V_N}{\partial x}\boldsymbol{G}(x) \qquad (16)$$

Using (4), (13) and (16), equation (15) can be rewritten as

$$\dot{V} = \frac{\partial V_N}{\partial x}[\boldsymbol{F}(x) + \boldsymbol{G}(x)\boldsymbol{u}_N] + \psi(x)[\boldsymbol{u}_c + \boldsymbol{\eta}(x) + \boldsymbol{f}(x)]$$
$$+ \frac{1}{\xi_1}t_r(\widetilde{\boldsymbol{W}}^T\dot{\hat{\boldsymbol{W}}}) + \frac{1}{\xi_2}(\hat{\tau} - \tau^*)\dot{\hat{\tau}}$$
$$\le -\alpha(\|x\|) + \boldsymbol{\varphi}^T(x)[\boldsymbol{u}_c + \boldsymbol{\eta}(x) + (\hat{\boldsymbol{W}} - \widetilde{\boldsymbol{W}})^T\boldsymbol{\Gamma} + \boldsymbol{J}(x)]$$
$$+ \frac{1}{\xi_1}t_r(\widetilde{\boldsymbol{W}}^T\dot{\hat{\boldsymbol{W}}}) + \frac{1}{\xi_2}(\hat{\tau} - \tau^*)\dot{\hat{\tau}} \qquad (17)$$

where $\hat{f}(x,\boldsymbol{W}^*) = \boldsymbol{W}^{*T}\boldsymbol{\Gamma} = (\hat{\boldsymbol{W}} - \widetilde{\boldsymbol{W}})^T\boldsymbol{\Gamma}$ and $\boldsymbol{\varphi}(x) = \boldsymbol{\psi}^T(x)$. Noting in (17) that

$$t_r(\widetilde{\boldsymbol{W}}^T\dot{\hat{\boldsymbol{W}}}) = \sum_{k=1}^{n_R}\widetilde{\boldsymbol{w}}_k^T\dot{\hat{\boldsymbol{w}}}_k \qquad (18)$$

$$\boldsymbol{\phi}^T\widetilde{\boldsymbol{W}}^T\boldsymbol{\Gamma} = \sum_{k=1}^{n_R}\boldsymbol{\phi}^T\widetilde{\boldsymbol{w}}_k b_k \qquad (19)$$

where $\widetilde{w}_k$ and $\dot{\hat{w}}_k^T$ are the kth columns of matrixces $\widetilde{W}$ and $\dot{\hat{W}}$, respectively. Then, (17) becomes

$$\dot{V} \le -\alpha(\|x\|) + \boldsymbol{\phi}^T(x)[\boldsymbol{u}_c + \boldsymbol{\eta}(x) + \boldsymbol{J}(x) + \hat{\boldsymbol{W}}^T\boldsymbol{\Gamma}]$$
$$+ \sum_{k=1}^{n_R}[\frac{1}{\xi_1}\widetilde{\boldsymbol{w}}_k^T\dot{\hat{\boldsymbol{w}}}_k - b_k\widetilde{\boldsymbol{w}}_k\boldsymbol{\phi}] + \frac{1}{\xi_2}(\hat{\tau} - \tau^*)\dot{\hat{\tau}} \qquad (20)$$

The corrective controller is chosen as

$$\boldsymbol{u}_c = -\hat{\boldsymbol{W}}^T\boldsymbol{\Gamma} - \hat{\tau}\,\mathrm{sgn}[\boldsymbol{\phi}(x)] \qquad (21)$$

in which

$$\dot{\hat{\boldsymbol{w}}}_k = \xi_1 b_k\boldsymbol{\phi}(x) \qquad (22)$$

$$\dot{\hat{\tau}} = \xi_2\boldsymbol{\phi}^T(x)\,\mathrm{sgn}[\boldsymbol{\phi}(x)] \qquad (23)$$

where $\mathrm{sgn}[\boldsymbol{\phi}(x)] = \mathrm{sgn}[\boldsymbol{\phi}_1(x),...,\boldsymbol{\phi}_i(x),...,\boldsymbol{\phi}_m(x)]^T$ and sgn denotos the sign function.

By substituting (21), (22) and (23) into (20), yields

$$\dot{V} \le -\alpha(\|x\|) + \boldsymbol{\phi}^T(x)\{\boldsymbol{\eta}(x) + \boldsymbol{J}(x) - \hat{\tau}\,\mathrm{sgn}[\boldsymbol{\phi}(x)]\} + (\hat{\tau} - \tau^*)\boldsymbol{\phi}^T(x)\,\mathrm{sgn}[\boldsymbol{\phi}(x)]$$
$$= -\alpha(\|x\|) + \boldsymbol{\phi}^T(x)\{\boldsymbol{\eta}(x) + \boldsymbol{J}(x) - \tau^*\,\mathrm{sgn}[\boldsymbol{\phi}(x)]\}$$
$$= -\alpha(\|x\|) + \sum_{i=1}^{m}\{\varphi_i(x)[\eta_i(x) + \vartheta_i(x) - \tau^*\,\mathrm{sgn}[\varphi_i(x)]\}$$
$$\le -\alpha(\|x\|) - \sum_{i=1}^{m}(\tau^* - (|\eta_i(x)| + |\vartheta_i(x)|)|\varphi_i(x)|)$$
$$\le 0 \qquad (24)$$

where ϑ_i is the ith element of $\boldsymbol{J}$.

IV. SIMULATION RESULT

Consider the simulations of a three-tank compression system. The input spaces for these examples are set between [-2.7 2.7]. The receptive-field basis functions are chosen as $\boldsymbol{\phi}_{jk}(s_j) = \exp[-(s_j - m_{jk})^2 / \boldsymbol{\sigma}_{jk}^2]$ for $j = 1, 2$ and $k = 1 - 12$. Here, the parameters are chosen as $\boldsymbol{\sigma}_{jk} = 2.4$ and $[m_{j1}, m_{j2}, m_{j3}, m_{j4}, m_{j5}, m_{j6}, m_{j7}, m_{j8}, m_{j9}, m_{j10}, m_{j11}, m_{j12}]$

$= [-3.3, -2.7, -2.1, -1.5, -0.9, -0.3, 0.3, 0.9, 1.5, 2.1, 2.7, 3.3]$ for all j and k. The initial values of the adjustable weights w_{kq} for $k = 1 - 12$, $q = 1, 2$ are all set

as zero. The parameters of CMAC are set as $n_E = 9$ and $\rho = 4$.

Considering a three-tank system as shown in Fig. 2, it's dynamic function is given as [1]

$$\dot{x}_1 = \frac{(-c_1 S_p \, \text{sgn}(x_1 - x_3)\sqrt{2g|x_1 - x_3|} + u_1)}{A} + \eta_1(x, u) \quad (25)$$

$$\dot{x}_2 = \frac{(-c_3 S_p \, \text{sgn}(x_2 - x_3)\sqrt{2g|x_2 - x_3|} - c_2 S_p \sqrt{2gx_2} + u_2)}{A}$$
$$- q_{20} + \eta_2(x, u) \quad (26)$$

$$\dot{x}_3 = \frac{(-c_1 S_p \, \text{sgn}(x_1 - x_3)\sqrt{2g|x_1 - x_3|} - c_3 S_p \, \text{sgn}(x_3 - x_2)\sqrt{2g|x_3 - x_2|})}{A}$$
$$+ \eta_3(x, u) \quad (27)$$

where $x = [x_1, x_2, x_3]^T$, $u = [u_1, u_2]^T$, g is the gravity acceleration, η_i, for $i = 1, 2, 3$ represent the modeling uncertainty due to the inaccuracy on the cross section of connection pipes. $q_{20} = c_1 S_P \sqrt{2gx_2}$ is the outflow rate from the tank 2. The cross section $A = 0.0154 \text{m}^2$ and the cross section of the connection pipes is $S_p = 5 \times 10^{-5} \text{m}^2$. The $c_1 = 1$, $c_2 = 0.8$ and $c_3 = 1$ denote the nondimensional outflow coefficients. The discrete time model is derived by using the forward Euler approximation $\dot{x}_i \cong (x_i(k+1) - x_i(k))/\Delta t$, for $i = 1, 2, 3$ where $\Delta t = 0.1s$ represents the sampling period. Then the discrete-time format can be obtained as
$x_i(k+1) = F_i(.) + G_i(.)[u(k) + \beta_i(.)f_i(.) + \eta_i(.)]$,
for $i = 1, 2, 3$ (28)
where f_i denotes the fault function and β_i is the time profiles of fault. The nominal controller is given as [12]

$$u_{iN} = \frac{\{[x_{id}(k+1) - x_{id}(k) + x_i(k)/\Delta t] + ax_{id}(k+1)\}(a + 1/\Delta t)^{-1} - F_i(.)}{g}$$
, for $i = 1, 2$ (29)

where $a = 10$. Initial condition is set to be the liquid levels $x_1(0) = x_2(0) = x_3(0) = 0.15\text{m}$, and the control objective is to keep all the liquid levels at 0.2m (i.e. $x_{1d}(k) = x_{2d}(k) = x_{3d}(k) = 0.2m$).

For simulation purpose, the modeling uncertainty bounds are set as $|\eta_i(x, t)| \leq \overline{\eta}_i$, where $\overline{\eta}_1 = 3.5 \times 10^{-3}$, $\overline{\eta}_2 = 2.5 \times 10^{-3}$ and $\overline{\eta}_3 = 6.5 \times 10^{-3}$. Three simulation cases are illustrated:

Case 1: The x_1, x_2 and x_3 are in the regular condition with $f_1(k) = 0$ and $f_2(k) = 0$ and use the control input $u_i = 0$, for $i = 1, 2$. The simulation results are shown in Fig. 3.

Case 2: The x_1, x_2 and x_3 are in the regular condition with $f_1(k) = 0$ and $f_2(k) = 0$ and use the control input $u_i = u_{iN}$, for $i = 1, 2$. The simulation results are shown in Fig. 4.

Case 3: Considering an abrupt leakage in tank 1 and an incipient leakage in tank 2, whose failure dynamics are [1]
$f_1(k) = -c_1 \pi \gamma_1^2 \sqrt{2gx_1(k)}$
$\beta_1(k - T_1) = U(k - T_1), \quad T_1 = 270$
$f_2(k) = -c_2 \pi \gamma_2^2 \sqrt{2gx_2(k)}$
$\beta_2(k - T_2) = (1 - e^{-\alpha(k - T_2)})U(k - T_2),$
$\alpha = 0.063; T_2 = 426$

where $\gamma_1 = 7.3 \times 10^{-2}$ and $\gamma_2 = 8.4 \times 10^{-2}$. Using the control inputs $u_i = u_{iN} + u_{iC}$, for $i = 1, 2$, the simulation results are shown in Fig. 5.

The simulation figures show the trajectories of x_1, x_2, x_3, and the control inputs u_1 and u_2. As illustrated in Fig. 5, the fault accommodation scheme is effective in correcting the effect of the unknown fault by attaching the corrective controller to the nominal controller.

V. Conclusions

A fault accommodation method using the cerebellar model articulation controller is proposed. The fault accommodation scheme is derived based on Lyapunov function, so that the accommodation control system is guaranteed to be stable even in the present of faults. The proposed fault accommodation control system is applied to a tank control system. Simulation results show that the proposed design method can effectively achieve the fault accommodation.

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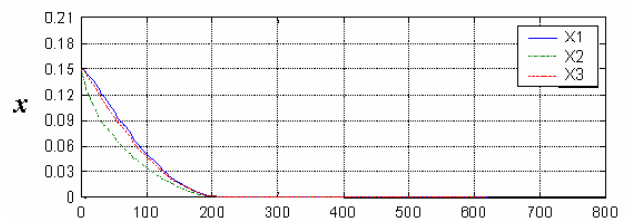
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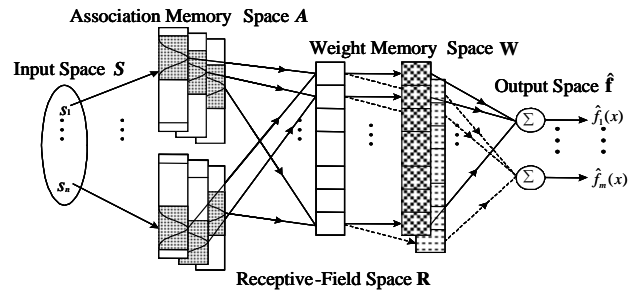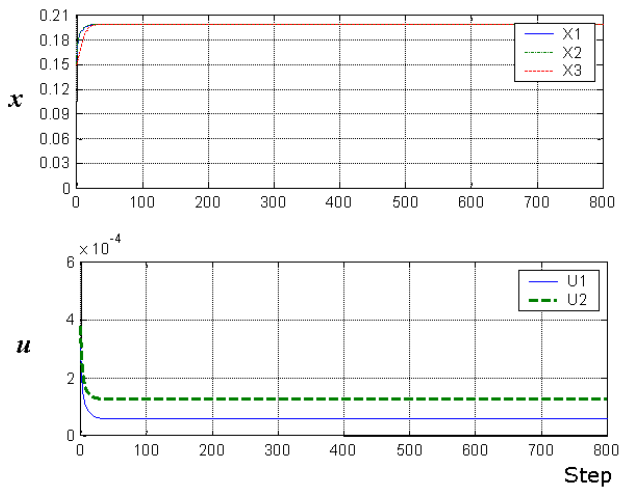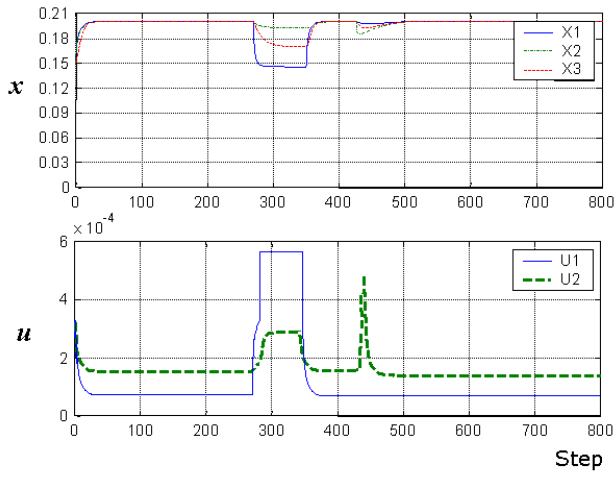
Fig. 1 Architecture of a CMAC
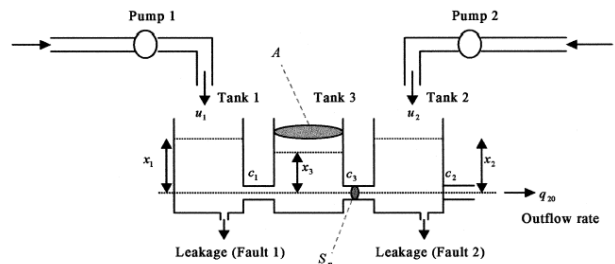
Fig. 2. Three-tank system

Fig. 3. Tank system without fault, $u_i = 0$ ($x1$ —, $x2$ ---, $x3$ − − −)

Fig. 4. Tank system without fault, $u_i = u_{iN}$,

Fig. 5. Tank system with fault occurring at
$T_1 = 270, T_2 = 426$, $u_i = u_{iN} + u_{iC}$,
$(x1 —, x2 ---, x3 - - -, u1 —, u2 ---)$