Sequential Antenna Selection for Diversity Combining over Nakagami Fading Channels

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Abstract

A sequential selection combining (SQC) scheme based on signal detection is proposed and analyzed for dual-branch space diversity over Nakagami fading channels. The complexity of the SQC is lower than the maximal-ratio combining (MRC). The SQC can be shown to have a better performance than the selection combining (SC), where the BPSK is considered as an example for illustration and its average bit error rate (BER) is derived and evaluated. Numerical results are presented to demonstrate the SSC performance.

I. Introduction

Space diversity with dual reception antennas is a valuable technique to enhance the receiver performance for wireless communications [1]-[3]. With space diversity, the received signals of the dual antenna branches may be linearly combined to combat the wireless fading effects [4].

In different linear combining methods, the conventional MRC yields a minimum symbol error probability for the signal detection at the receiver site. For coherent modulation, although the MRC yields an optimal detection performance, its implementation involves the coherent addition of dual-branch signals, which is power-consuming and not cost-effective, and thus impractical for commercial applications.

On the other hand, the dual-branch SC utilizes the branch (say branch 1) with a larger SNR for signal detection. The SC is suboptimal but its implementation complexity is much lower than that of the MRC. However, with the pure SC, the information from the other branch (i.e. branch 2) with a lower SNR is wasted. In this sense, the performance of the SC can be enhanced by using the information collected from branch 2. At the selection stage in the SC scheme, since the signals and channel estimations of dual-branches are already obtained, the information from both branches should be used for following signal detection.

In this paper, based on the above viewpoint, the conventional SC is improved with signal detection on wireless fading channels.

II. SQC and Fading Channel Models

In the context, the discrete-time model is employed. Thus, for a transmitted symbol s_m (m=0,1), the received signal of diversity branch l (l=1,2) at any discrete-time instance is

$$r_l = \alpha_l s_m + n_l, \quad m = 0,1 \tag{1}$$

where α_l is the fading factor with the probability density function (PDF) denoted by $f_l(\cdot)$, and n_l is the additive Gaussian noise with a zero mean and variance $N_0/2$ [5].

For BPSK modulation, $s_1 = \sqrt{E_b}$ and $s_0 = -\sqrt{E_b}$ may be used, where E_b is transmitted signal energy. Throughout the paper, the independent Nakagami fading channels are considered to model the dual wireless channels.

Let $\gamma_l = \alpha_l^2 E_b / N_0$ be the faded SNR of branch l $(l = 1, 2)$. For the Nakagami fading channel, the PDF of γ_l $(l = 1, 2)$ is given by [6]

$$f_l(\gamma_l) = \frac{m_l^{m_l} \gamma_l^{m_l - 1}}{\bar{\gamma}_l^{m} \Gamma(m_l)} e^{\frac{-m_l \gamma_l}{\bar{\gamma}_l}}, \quad \gamma_l \geq 0 \tag{2}$$

where $m_l \geq 1/2$ represents the fading severity and $\bar{\gamma}_l = E[\gamma_l]$. When $m_l = 1$, (2) also characterizes the PDF of Rayleigh fading factor. Let $f_{12}(\cdot,\cdot)$ denote the joint PDF of (γ_1, γ_2). For the independent Nakagami fading channels, $f_{12}(\gamma_1, \gamma_2) = f_1(\gamma_1) f_2(\gamma_2)$.

According to the MRC and/or the MAP detector, under fixed (α_1, α_2), the decision variable for signal detection is given by $\alpha_1 r_1 + \alpha_2 r_2$, and the corresponding decision test is [5]

$$\alpha_1 r_1 + \alpha_2 r_2 \underset{s_0}{\overset{s_1}{\gtrless}} 0. \tag{3}$$

Now, suppose that the branch with a larger SNR (say branch one) is first selected for signal detection, and assume that $\eta_2 = -\alpha_2 r_2$ is already known for the time being, then the test given by (3) becomes

$$\alpha_1 r_1 \underset{s_0}{\overset{s_1}{\gtrless}} \eta_2 \tag{4}$$

where η_2 functions as a test threshold for the signal detection. Thus, although only one diversity branch is used as in the SC, for the above signal detection problem, the optimal decision test should not be the conventional one in which the signal of the selected branch is only compared to a zero threshold. If a non-zero test threshold can be set in some meaningful way, the detection performance will be better than that of the conventional SC, but with a lower implementation complexity than the MRC.

With $\eta_2 = -\alpha_2(\alpha_2 s_m + n_2)$, (4) can be rewritten as

$$\alpha_1 r_1 + \alpha_2^2 s_m \underset{s_0}{\overset{s_1}{\gtrless}} -\alpha_2 n_2 \tag{5}$$

where $\alpha_1 r_1 + \alpha_2^2 s_m$ is the test statistic and $-\alpha_2 n_2$ is the threshold analogy. Two observations are made here. First, for the decision variable given in (5) we have

$$\alpha_1 r_1 + \alpha_2^2 s_1 \geq \alpha_1 r_1 + \alpha_2^2 s_0 \tag{6}$$

Second, under fixed α_2 (i.e. with channel estimation in the practical diversity combining), $-\alpha_2 n_2$ in (5) has a zero mean. Consequently, if branch l is selected on the basis of $\gamma_l > \gamma_{\bar{l}}$ with $\bar{l} = 3 - l$, then a reasonable decision test based on (5) and (6) should be

$$\alpha_l r_l + \begin{cases} \alpha_{\bar{l}}^2 s_0 > ^{s_1} \\ \alpha_{\bar{l}}^2 s_1 <_{s_0} \end{cases} 0. \tag{7}$$

Meanwhile, if

$$\alpha_l r_l + \alpha_{\bar{l}}^2 s_0 < 0 \quad \text{and} \quad \alpha_l r_l + \alpha_{\bar{l}}^2 s_1 > 0 \tag{8}$$

then another branch (i.e. $\bar{l}$) is used with the decision policy also given by (7) for signal detection. Nevertheless, if both ranges of the dual-branch signal are located in the region given by (8), the conventional SC is employed for signal detection.

An interesting notice is that the above signal detection policy also yields a new diversity combining method. In the new combining scheme, the receiver first selects the branch with a larger SNR for signal detection but with non-zero test thresholds $-\alpha_{\bar{l}}^2\sqrt{E_b}$ and $\alpha_{\bar{l}}^2\sqrt{E_b}$. If a detection decision cannot be made, the other branch with a lower SNR is then used with the two non-zero test thresholds. If the decision cannot be made either, the branch with a larger SNR is utilized with a zero test threshold for signal detection, where now the decision space becomes more specific than that of the pure SC because the dual-branch signals are already in smaller uncertain ranges in the signal space.

The SQC will outperform the conventional SC since the uncertain signal space is smaller, and since it does not use the coherent addition as in the MRC, it has less complexity than the MRC.

III. BER Evaluation

Let $y_{lj} = \alpha_l r_l + \alpha_{\bar{l}}^2 s_j$ for $l = 1, 2$ and $j = 0, 1$. Under fixed fading factors (α_1, α_2), if s_m has been transmitted, y_{lj} has a Gaussian distribution with the mean $\alpha_l^2 s_m + \alpha_{3-l}^2 s_j$ and the variance $\alpha_l^2 N_0 / 2$. Suppose branch one is first selected with $\gamma_1 \geq \gamma_2$. With the SQC, under the situation that s_0 has been transmitted, the BER can be evaluated by

$$P_{0,\gamma_1>\gamma_2}^{(dsc)}(\gamma_1,\gamma_2) = \Pr(y_{10}>0,\gamma_1>\gamma_2 \mid s_0) + \Pr(y_{20}>0, y_{10}<0, y_{11}>0, \gamma_1>\gamma_2 \mid s_0) + \Pr(r_1>0, y_{i0}<0, y_{i1}>0, i=1,2, \gamma_1>\gamma_2 \mid s_0) \tag{9}$$

where $\operatorname{erfc}(\cdot)$ represents the complementary error function,

$$\Pr\left(y_{10}>0,\gamma_1>\gamma_2, s_0\right) = \frac{1}{2}\operatorname{erfc}\left(\frac{\gamma_1+\gamma_2}{\sqrt{\gamma_1}}\right)_{\gamma_1>\gamma_2}, \tag{10}$$

$$\Pr(y_{20}>0, y_{10}<0, y_{11}>0, \gamma_1>\gamma_2 \mid s_0) = \frac{1}{4}\operatorname{erfc}\left(\frac{\gamma_1+\gamma_2}{\sqrt{\gamma_2}}\right)_{\gamma_1>\gamma_2}\left[\operatorname{erfc}\left(\frac{\gamma_1-\gamma_2}{\sqrt{\gamma_1}}\right)_{\gamma_1>\gamma_2} - \operatorname{erfc}\left(\frac{\gamma_1+\gamma_2}{\sqrt{\gamma_1}}\right)_{\gamma_1>\gamma_2}\right], \tag{11}$$

and

$$\begin{aligned}&\Pr\left(r_1>0, y_{i0}<0, y_{i1}>0, i=1,2, \gamma_1>\gamma_2 \mid s_0\right)\\ &= \frac{1}{4}\left[\operatorname{erfc}\left(\sqrt{\gamma_1}\right)_{\gamma_1>\gamma_2} - \operatorname{erfc}\left(\frac{\gamma_1+\gamma_2}{\sqrt{\gamma_1}}\right)_{\gamma_1>\gamma_2}\right]\times\left[\operatorname{erfc}\left(\frac{\gamma_2-\gamma_1}{\sqrt{\gamma_2}}\right)_{\gamma_1>\gamma_2} - \operatorname{erfc}\left(\frac{\gamma_1+\gamma_2}{\sqrt{\gamma_2}}\right)_{\gamma_1>\gamma_2}\right].\end{aligned} \tag{12}$$

Similarly, if branch two is selected with $\gamma_1 \leq \gamma_2$, the BER under the situation that s_0 has been transmitted

$$\begin{aligned}P_{0,\gamma_1\leq\gamma_2}(\gamma_1,\gamma_2) &= \frac{1}{2}\operatorname{erfc}\left(\frac{\gamma_1+\gamma_2}{\sqrt{\gamma_1}}\right)_{\gamma_1\leq\gamma_2} + \frac{1}{4}\operatorname{erfc}\left(\frac{\gamma_1+\gamma_2}{\sqrt{\gamma_2}}\right)_{\gamma_1\leq\gamma_2}\times\left[\operatorname{erfc}\left(\frac{\gamma_2-\gamma_1}{\sqrt{\gamma_1}}\right)_{\gamma_1\leq\gamma_2} - \operatorname{erfc}\left(\frac{\gamma_1+\gamma_2}{\sqrt{\gamma_1}}\right)_{\gamma_1\leq\gamma_2}\right]\\ &+ \frac{1}{4}\left[\operatorname{erfc}\left(\sqrt{\gamma_1}\right)_{\gamma_1\leq\gamma_2} - \operatorname{erfc}\left(\frac{\gamma_1+\gamma_2}{\sqrt{\gamma_1}}\right)_{\gamma_1\leq\gamma_2}\right]\times\left[\operatorname{erfc}\left(\frac{\gamma_1-\gamma_2}{\sqrt{\gamma_2}}\right)_{\gamma_1\leq\gamma_2} - \operatorname{erfc}\left(\frac{\gamma_1+\gamma_2}{\sqrt{\gamma_1}}\right)_{\gamma_1\leq\gamma_2}\right]\end{aligned} \tag{13}$$

For equally probable symbols, the final average BER can be evaluated as

$$\overline{P}_b = \frac{1}{2}\int_0^{\infty}\int_0^{\gamma_1}\left(P^{(sqc)}_{0,\gamma_1>\gamma_2}(x,y) + P^{(sqc)}_{1,\gamma_1>\gamma_2}(x,y)\right) f_1(x)\, f_2(y)dydx$$
$$+\frac{1}{2}\int_0^{\infty}\int_0^{\gamma_2}\left(P^{(sqc)}_{0,\gamma_1\le\gamma_2}(x,y) + P^{(sqc)}_{1,\gamma_1\le\gamma_2}(x,y)\right) f_1(x)\, f_2(y)dxdy. \quad (14)$$

The average BER given by (14) can be easily obtained by using computation tools such as MATLAB.

IV. Numerical Results and Conclusions

For numerical evaluations, without loss of generality, the balanced case of $\overline{\gamma}_1 = \overline{\gamma}_2 = \overline{\gamma}$ is considered to alleviate parameter assignments. In Fig. 1, the average BER for different values of *m* is presented, where the case of $m=1$ is for the Rayleigh fading channel. From the numerical result, the SQC has a lower average BER than the SC for the Rayleigh and Nakagami fading channels. The numerical result also validates the BER comparison presented in the above analysis. In addition, the SQC performance over other fading channel models can also be examined in the same way.

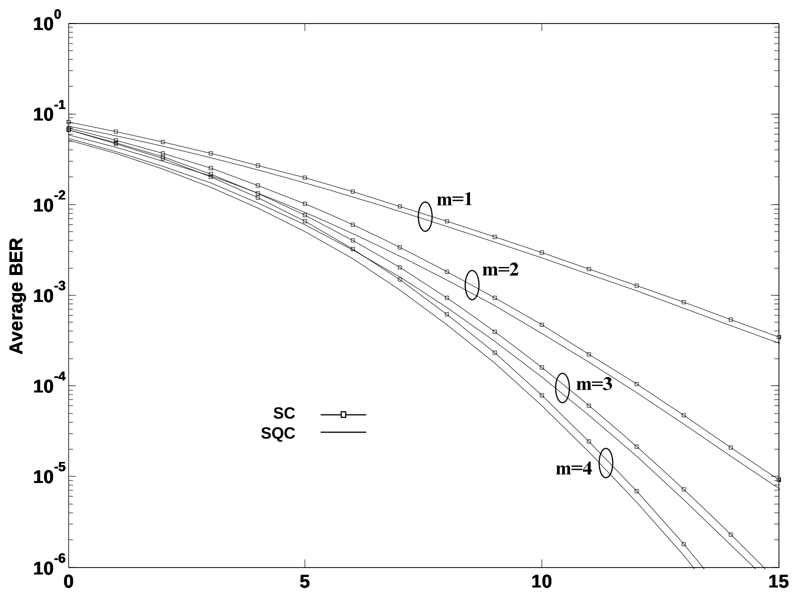

Fig. 1 Average BER for the SQC and SC.

References:

[1]W. C. Jakes, *Microwave Mobile Communication*, 2nd ed. Piscataway, NJ: IEEE Press, 1994.

[2]A. F. Molisch and M. Z. Win, "MIMO systems with antenna selection," *IEEE Microwave Mag.*, vol. 5, pp. 46–56, Mar. 2004.

[3]H. Furukawa, K. Hamabe, and A. Ushirokawa, "SSDT-site selection diversity transmission power control for CDMA forward link," *IEEE J. Sel. Areas Commun.*, vol. 18, pp. 1546–1553, Aug. 2000.

[4]D. G. Brennan, "Linear diversity combining techniques," *Proc. IRE*, pp.1075-1102, June 1959.

[5]J. G. Proakis, *Digital Communications*, 4th ed. New York: McGraw-Hill, 2001.

[6]M. K. Simon, M.-S. Alouini, *Digital Communication over Fading Channels,* John Wiley & Sons, Inc., 2005.