A Novel Approach for Inventory Classification in Two-Stage Supply Chain System

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***Abstract*—This research applied Selective Regeneration Particle Swarm Optimization (SRPSO) for two-stage supply chain inventory classification problems. In SRPSO, In order to increase the efficiency, suggestion on parameter settings is made and a mechanism is designed to prevent particles fall into the local optimal. This research is assumed that an inventory classification is performed to properly categorize items into a number of groups. Items in the same group are then jointly replenished. That is, items in the same group have the same order interval at the warehouse and the retailer. The goal of this study is to classify a known set of items into a number of groups and determine the optimal order interval of each classification group to minimize the total relevant cost of the supply chain system. This study tests a real dataset and two article datasets to compare the results to other known classification and non-classification methods. The outcomes fully demonstrate that SRPSO is an efficient, accurate, and robust method for inventory classification in supply chain problems. The SRPSO performs comparatively better than other grouping and non-grouping techniques.**

Keywords-Particle Swarm Optimization; Selective Regeneration Particle; Inventory Classification

I. INTRODUCTION

As an economy grows, members in a supply chain such as manufacturers, distributors, warehouses and retailers may carry hundreds to thousands of individual items. Managing multiple item inventories in supply chain systems is becoming increasingly more complex and difficult. Effective multi-item inventory management methods are necessary for better supply chain management. Among these management methods is inventory classification. In this study, an improved PSO will be proposed and designed by two mechanisms. A suggestion on the setting of cognition and social parameters is proposed to accelerate convergence. Furthermore, a selective particle regeneration mechanism is designed for avoiding the search trapped in local optima. The improved PSO will be applied for inventory classification problem in a two stage supply chain. Nowadays, PSO has been widely applied in many research areas and real-world engineering fields, such as, task assignment and scheduling [1], roundness measurement [2], demand forecast [3], financial decisions [4], product plans [5] and layout design [6][7] etc. Usually PSO is considered because of the ease of implementation and effectiveness. It can solve continuous problem and obtain the good performance. Though it has been demonstrated that PSO performs well in many optimization problems, it was observed that the algorithm did not perform well at times.

II. PROBLEM

This chapter considers a supply chain system that contains a warehouse and a retailer. The retailer receives and fills demand for multiple items from outside customers with its inventory. The retailer replenishes its inventory by placing orders with the warehouse. The warehouse fills those orders with its own inventory. Inventory at the warehouse is replenished from outside suppliers who are assumed to have unlimited capacity. An order placed by the warehouse or the retailer may contain more than one item. A main ordering cost occurs when the warehouse or the retailer places an order that is independent of the number of items included in this order. In the meantime, a setup cost occurs for an item included in the order. Other considered costs related to inventory control include costs of carrying inventory at the warehouse and the retailer, respectively.

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A. Notation

M: number of classification groups.
N: number of items.
T_j: order interval of classification group j.
A_w: main ordering cost for orders placed by the warehouse.
A_r: main ordering cost for orders placed by the retailer.
a_{wi}: setup cost of item i at the warehouse.
a_{ri} : setup cost of item i at the retailer.
r: cost of carrying one dollar of the item in inventory for a unit time interval.
v_{wi}: variable cost of item i at the warehouse.
v_{ri}: variable cost of item i at the retailer.
S_j: set of the items in classification group j.
D_i: demand per unit time of item i.

The various cost of this paper can be thoroughly explained as:

- Main Ordering Cost (A): The main ordering cost component incurred with each replenishment. It includes the cost of order forms, postage, telephone calls, authorization, typing and of order, receiving, inspection, following up on unexpected situations and handing of vendor invoices.
- Setup Cost (a): Expenses incurred in setting up a machine, work center, or assembly line, to switch from one production job to the next and location of machinery, and employee hiring and training.
- Carrying Cost (r): The carrying charge, the cost having one dollar of the item tied up of in inventory for a unit time interval. It includes the opportunity cost of money invested, the expense incurred in running a warehouse, handling and counting cost, the cost of special storage requirement, deterioration of stock, damage, theft, obsolescence, insurance and taxes.
- Variable Cost (v): The unit value of an item is expressed in dollars per unit. For a merchant it is simply the price paid to supplier, plus any cost incurred to make it ready to sale.

B. Objective Functions

Tsai and Yeh [8] consider the main ordering costs in their study. They assumed. When the retailer places an order for items of classification group j, the incurred main ordering cost and setup costs can be expressed by (1).

$$\text{Ordering cost and Setup} = A_r + \sum_{i\in S_j} a_{ri} \tag{1}$$

Average inventory level of item i in group j is $(T_j D_i)/2$. Therefore, total relevant costs of the items in group j at the retailer can be written as (2).

$$\text{TRC in Retailer} = \frac{A_r \sum_{i\in S_j} a_{ri}}{T_j} + \frac{1}{2} T_j \sum_{i\in S_j} D_i v_{ri} r \tag{2}$$

A similar expression can be derived for the total relevant costs of the items in group j at the warehouse. As a result, total relevant costs of the system can be written as (3).

$$\text{TRC} = \sum_{j=1}^{M} \left[\left(\frac{\mathrm{A_w} + \sum_{i\in S_j} a_{wi}}{T_j} + \frac{1}{2} T_j \sum_{i\in S_j} D_i v_{wi} r \right) + \left(\frac{\mathrm{A_R} + \sum_{i\in S_j} a_{ri}}{T_j} + \frac{1}{2} T_j \sum_{i\in S_j} D_i v_{ri} r \right) \right] \tag{3}$$

It can simply be derived the formula for the optimal order interval of classification group j as (4).

$$T_j = \sqrt{\frac{2(A_w + \sum_{i\in S_j} a_{wi} + A_r + \sum_{i\in S_j} a_{ri})}{\sum_{i\in S_j} D_i v_{wi} r + \sum_{i\in S_j} D_i v_{ri} r}} \tag{4}$$

III. METHODOLOGY

Social behavior observed in flocks of birds and schools of fish has inspired Particle swarm optimization (PSO) [9]. In nature, most members follow a leader who leads the bird or fish group to move. In PSO, a particle represents a potential solution to the considered problem, similar to the individuals in the bird and fish group. Each particle travels in the solution space and attempts to move toward a better solution by changing its direction and speed based on its experience and information from the current best particle of the swarm.

In general, the procedure of PSO is described as follows:

(1) Particle Initialization: An initial swarm of particles is generated in the search space. Usually, the population size is decided by the dimension of problems.

(2) Velocity and Position Update**:** In each iteration, a new velocity value for each particle is calculated based on its current velocity, the distance from its previous best position, and the distance from the global best position. The new velocity value is used to calculate the next position of the particle in the search space. The particle's velocity and position are dynamically updated as follows:

$$V_{id}^{new} = w \times V_{id}^{old} + c_1 \times rand \times (P_{id} - x_{id}^{old}) + c_2 \times rand \times (P_{gd} - x_{id}^{old}) \tag{5}$$

$$x_{id}^{new} = x_{id}^{old} + V_{id}^{new} \tag{6}$$

The new velocity of a particle, V_{id}^{new}, is updated by (5), considering the particle's previous velocity, V_{id}^{old}, and previous position, x_{id}^{old}. $w = [0.5 + rand\ /\ 2]$ is an inertia weight and rand is a uniformly generated random number between 0 and 1. The cognition parameter, c_1, and social parameter, c_2, are acceleration coefficients that are conventionally set to a fixed value 0 to 2.0. P_{id} is the previous individual best position of this particle and P_{gd} is the current global best position. (6) then calculates the new position of the particle, x_{id}^{new}.

(3) Evaluation and Update of Best Locations**:** The fitness value of each particle is calculated by the objective function. The values of P_{id} and P_{gd} are evaluated and replaced if a better particle best position or global best position is obtained.

(4) Termination**:** Step (2) and step (3) are repeated iteratively until the termination condition is met.

A. Selective Regenerated Particle Swarm Optimization

Tsai and Kao [10] proposed selective regenerated particle swarm optimization (SRPSO) to improve the accuracy and efficiency of PSO and designed two new features. First, a suggestion on the setting of cognition and social parameters, c_1 and c_2, is proposed to accelerate convergence. Furthermore, the mechanism of selective particle regeneration is designed for avoiding the search trapped in local optima.

B. Cognitive and Social Parameter Setting

As shown in (1), the new velocity of a particle is determined based on the best individual position and the knowledge of the swarm's best. ($P_{id} - x_{id}^{old}$) represents the cognitive knowledge and ($P_{gd} - x_{id}^{old}$) corresponds to the social knowledge. Their relative effect on the new velocity is determined based on the respective weights of parameter c_1 and c_2. As recommended by Kennedy and Eberhart, the two parameters are typically assigned the same value. As a result, the next position of a particle is typically the middle of P_{id} and P_{gd}. It may take more iteration for the particle to move closer to the global best location and thus affect the efficiency of the search.

To accelerate convergence, the setting suggests assigning a larger value to c_2 with respect to c_1. Consequently, the new particle position will be closer to the global best location, P_{gd}. The suggested setting accelerates particle convergence.

C. Selective Regenerated Particle Swarm Optimization

The suggested parameter setting that c_2 is greater than c_1, may be able to improve the efficiency of convergence, but it also increases the risk of particles falling into local optimum. Therefore, a "Selective Particle Regeneration" mechanism is designed. It is a new operation in which is similar to the mutation mechanism in GA. Generally speaking, as a particle becomes closer to local optimal location, the possibility of this particle escaping from it decreases, especially with the suggested parameter setting. The "Selective Particle Regeneration" mechanism first computes the distance, in terms of fitness value, between a particle and global best particle (P_{gd}). For particles with distances to the global best particle smaller than a predetermined value, f , d % of these particles will be randomly selected and regenerated.

The purpose of particle regeneration is to help some of the particles that are close to the global best particle escape from local optimum if the current global best particle represents a local optimal solution. However, the current global best particle may still contain valuable knowledge that may lead to better solutions. Therefore, partial knowledge carried by the global best particle will be adopted when generating new locations of the selected particles. More specifically, when determining the value of a specific dimension for the new location of a particle, the value of the same dimension of the current global best location is adopted with a probability of c. With a probability of $(1– c)$, the value is randomly generated.

Finally, it is desired for these regenerated particles not to move toward the global best particle right away. Therefore, as opposed to setting c_2 to be greater than c_1 as suggested previously, c_1 is given a value larger than c_2 instead when determining the new velocities of the regenerated particles. By doing so, greater weight is assigned to cognitive knowledge. This setting, however, applies only to the determination of velocities for particles that are just regenerated.

D. PSO and K-mean for inventory classification

This study chooses five item related characteristics as the vector for inventory classification. These include the main ordering cost at warehouse (A_w), retailer (A_r), variable cost at warehouse (v_w), retailer (v_r), and demand (D). The following describes the PSO and K-mean procedure for inventory classification.

(1) Solution generation: The potential solution to the considered problem is generated randomly. A solution includes the M centroid vector. In PSO, every potential solution is called a particle; every particle is an independent solution and improves in each generation. In K-mean, there is only one solution which be modified in each iteration.

(2) Item classification: The n items are grouped into M generated centroid vectors based on some similarity metric, which establishes a rule for assigning patterns to the domain of a particular centroid vector. This paper bases the criterion of inventory classification algorithm on the Euclidean distance calculation. Each item is grouped into the closest centroid vectors. The following determines the Euclidean distance of an item to the centroid vector.

$$D(x_p, z_j) = \sqrt{\sum_{i=1}^{M}(x_{pi} - z_{ji})^2} \tag{7}$$

where $\mathbf{x}_p$ is the p^{th} data vector, $\mathbf{z}_j$ is the centroid vector of group j, and M is the number of features of each centroid vector.

(3) Solution recalculation and improvement: K-mean recalculates the group centroid vector (solution), using:

$$\mathbf{z}_j = \frac{1}{n_j}\sum_{\forall \mathbf{x}_p \in C_j}\mathbf{x}_p \tag{8}$$

where n_j is the number of data vectors in group j and C_j is the subset of data vectors that form group j. Thus, the Euclidean distance of items to the centroid vector completely dominates the K-mean solution. PSO updates the particle position by (5) and (6). The objective function is to minimize total relevant costs as in (3).

(4) Stop: Step 2 to 3 of K-mean are repeated, until the centroid vector does not change. The PSO procedure is stopped when the termination condition is met.

Furthermore, PSO automatically classifies the inventory items to an optimal number of groups. In other words, users do not pre-determine the number of groups. First, a bigger and suitable number of groups are set. During the PSO evaluation, the fitness is evaluated and the number of groups is reduced to optimal in the process. Finally, the best total related cost and number of groups are obtained. The resulting group numbers are obtained, which are smaller or equal to the number of classification groups initially set.

IV. EPXPERIMENT SETTING AND DATASET

The experiment applied six algorithms to solve inventory classification problems, including four classification algorithms: SRPSO, PSO, K-mean, ABC classification, and two non-classification algorithms (NC1 and NC2). NC1 assumes that each item has a respective replenishment cycle. In another words, the number of items is the group number. NC2 assumes that all items are grouped in one group and have a joint replenishment cycle. In this study, all tested algorithms were coded in Matlab and run on a computer equipped with AMD 1.7G CPU and memory capacity of 1024 MB.

A. Dataset

A real dataset and three artificial datasets with a variety of complexity were used. Table I. summarizes the characteristics of the selected datasets, including main ordering (A), setup cost (a), variable cost (v), demand value (D), and the number of items (n).

Sport Product (S.P.) is a dataset collected from a famous golf equipment company. There are thirty items in this dataset. For further exploration, a dataset marked as Sport Product-Extended (S.P.EX) is created by expanding dataset S.P.. In addition, an artificial datasets are created for test. Dataset Art contains 500 items randomly generated based on Pareto principle.

TABLE I. Characteristics of five selected datasets

	n		v_w	v_r	D	a_w	a_r
S. P.	30	Max	5250	9750	70000	1055	1955
		Min	75	150	1900	20	35
S.P. EX	300	Max	6188	11323	79568	1251	2339
		Min	62	127	1626	17	28
Art	500	Max	1999	2983	8982	200	296
		Min	100	108	54	10	16

B. Parameter Setting

After thorough tests and experiments, we proposed the parameters setting for SRPSO in TABLE II. N is the dimension of benchmark function. F.E. is the function evaluation. It is also determined by N which is 50×N×Particle Size. The initial population is randomly generated. Both parameter c_1 and c_2 are 1.5 in PSO. Each test was performed 30 times for PSO and SRPSO. The termination condition is that the number of function evaluation is reached.

TABLE II. Parameter setting

Paraeter	c_1	c_2	d	f
Value	0.5	2.5	0.3	0.3
Parameter	a	Particle Size	F. E.	
Value	0.7	$5\times K \times dim$	$10\times M\times dim\times$Particle Size	

V. EXPERIMENT RESULTS AND ANALYSIS

This section evaluates four classification algorithms and two non-classifications algorithms and compares their performances. The current study also compares the quality of the respective classification and measures quality by the following two aspects:

(1) The total relevant cost (TRC): TRC is the objective function in this experiment as defined in (2). The measurement presents the best, average, and standard deviation value of TRC. Obviously, the smaller TRC has higher grouping quality. I.V. is improvement value, which is the difference between the TRC value and the best of each algorithm.

(2) Item classification: Observations show the relation between the classification result and item characteristics in each case. In other words, we would like to understand the effect of each characteristic for classification results and discuss the difference in classification result between different objective functions. C.N. is the number of groups.

- Sport Product

TABLE III illustrates the TRC from the six algorithms for the real dataset, Sport Product (S.P.). The SRPSO attains the lowest TRC value. PSO performs well when almost all outcomes are very close to SRPSO. The relative outcomes of ABC, K-mean, NC1, and NC2 are inferior to the heuristic algorithms in every aspect. Although ABC is a simple and old classification approach, the results still outdo those of non-classification. Fig. 1 and 2 show the characteristic value distribution of items in Sport Product for SRPSO, PSO, and K-mean. In this case, several items have extremely large demand value. Thus, no matter what the classification algorithm, these items dominate to classify by extremely large demand value. In SRPSO and PSO, when the demands of items are smaller, the demand and variable cost dominate classification results simultaneously. Altogether, the signal item characteristic does not decide most SRPSO and PSO outcomes. K-means is a traditional and effectual classification technique, but the solutions typically fall into the local optimal. This is because the objective function minimizes the standard Euclidean distance, which is mostly insufficient in forming groups. Thus, the extremely large value dominates the classification outcome. Fig. 2 illustrates the K-mean results in Sport Product. Grouping 2 and 4 only contain one and two items. In general, consolidating similar groups decreases ordering cost in a supply chain and, thus, we conclude that the extreme item characteristic in this real dataset fully dominates classification outcomes of K-means.

TABLE III. Comparison of TRC value in Sport Product

Sport Product						
	SRPSO	PSO	K-mean	ABC	NC1	NC2
Avg.	**2994340**	2995636	3133699	3047205	3496381	3048495
Best	**2993112**	**2993112**	3112628			
Std.	1585	**1162**	49076			
I.V.	0	0	119516	54093	503269	55383
C. N.	3	3	4	3	30	1

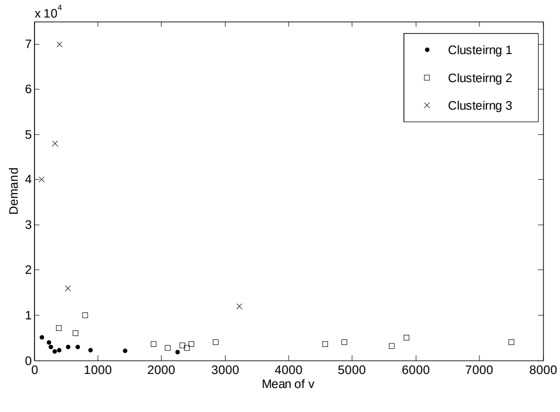

Figure 1. Classification for Sport Product by PSO and SRPSO

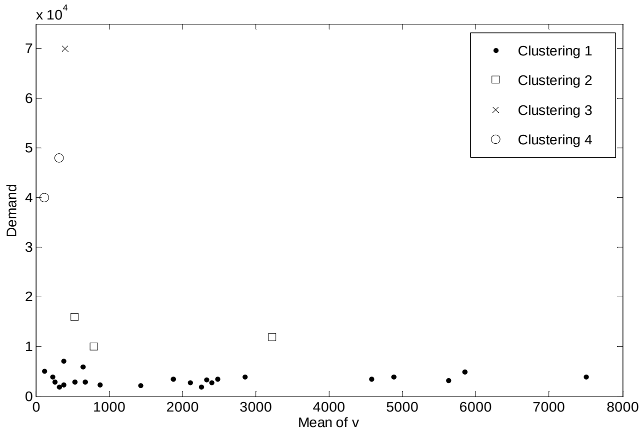

Figure 2. Classification for Sport Product by K-mean

- Spot Product-Extended (S.P.EX)

Table IV lists the TRC values obtained from the six methods on the dataset Sport Product-Extended. This bigger-sized dataset contains 300 items; thus, the resulting numbers of classification groups are larger no matter which classification technique is used. The table clearly shows that SRPSO dominates over the other methods. Noticeably, the outcomes of ABC classification are better than those of K-mean and the non-classification cases in both Sport Product and Sport Product-Extended. The K-mean yields higher TRC values and exhibits large variation. Furthermore, the four classification schemes have better performance than the two non-classification cases, showing the worth of inventory classification.

- Art

Table V lists the test results of the six algorithms for dataset Art, again showing the superiority of the proposed methods over PSO, K-mean, ABC, NC2, and NC1. For the best TRC values, SRPSO outperforms PSO slightly, and the standard deviations of SRPSO are significantly smaller than those of the other two methods. However, four classification algorithms all attain lower TRC values than the non-classification cases. Fig. 3, 4, 5, and 6 provide the classification results by the four classification algorithms. Compared to PSO, the boundaries between the groups by SRPSO are more sloping down to the right. It indicates that the effect of the demand factor and the variable cost factor are more balanced when applying SRPSO. The boundaries of item groups in Fig. 3 are very clear. Items are classified into four major groups based on demand values. When the demand is lower than 4,000, items are further split into two groups based on variable costs. It again shows values rather than item characteristics are the dominating factor under K-mean. Fig. 6 illustrates the result of ABC classification. Group A includes 10% of items that hold up about 50-60% of the total inventory money. Group B includes 20% of items that hold up about 20-30% of the total inventory money. The items not included in group A nor B are grouped under group C. The more sloping boundaries of item groups indicate the equal effect of the demand factor and variable cost factor, which may not align with the considered object of cost minimization.

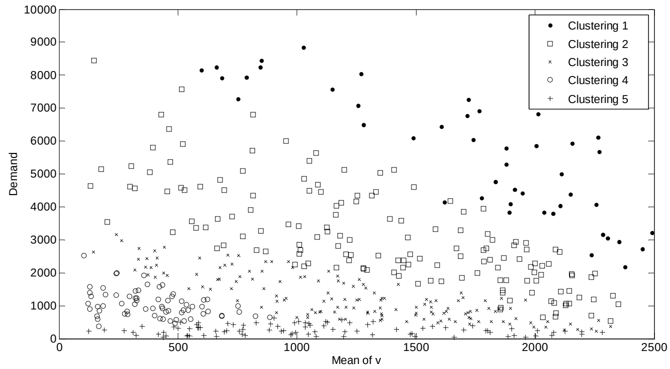

Figure 3. Classification for Art by SRPSO

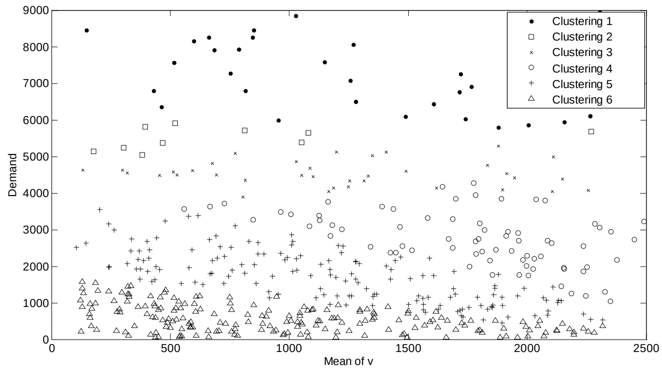

Figure 4. Classification for Art by PSO

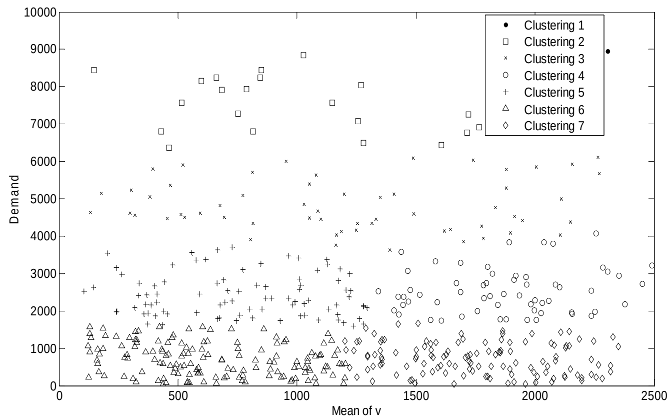

Figure 5. Classification for Art by K-mean

TABLE IV. Comparison of TRC value in Sport Product-Extened

Sport Product-Extened						
	SRPSO	PSO	K-mean	ABC	NC1	NC2
Avg.	**29610151**	29869697	30797229	30172872	32506969	30721358
Best	**29548472**	29795297	30485293			
Std.	**25103**	82013	366229			
I.V.	0	246825	936821	624400	2958497	1172886
C. N.	5	4	7	3	300	1

TABLE V. Comparison of TRC value in Art

Art						
	SRPSO	PSO	K-mean	ABC	NC1	NC2
Avg.	**11468915**	11512250	11709566	13101966	14900617	13148848
Best	**11435672**	11457987	11633277			
Std.	**18729**	44658	45655			
I.V.	0	22315	197605	1666294	3464945	1713176
C. N.	**5**	6	7	3	500	1

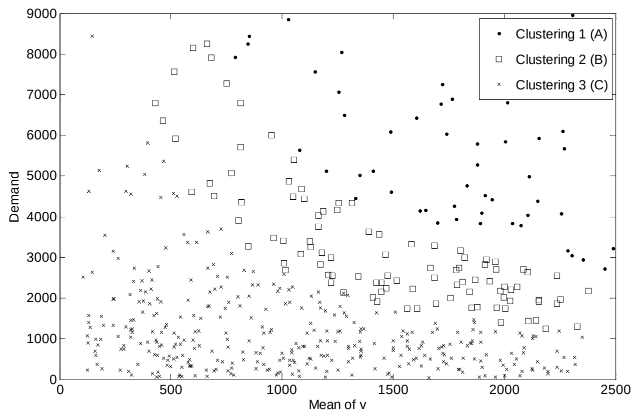

Figure 6. Classification for Art dataset by ABC

VI. CONCLUSION AND FUTURE WORK

This research develops a flexible inventory classification algorithm by applying Selective Regeneration Particle Swarm Optimization (SRPSO). This method classifies items to minimize total relevant costs of a supply chain system. Unlike most existing classification algorithms that need to specify the number of classification groups before classification, the proposed algorithm automatically determines the optimal number of groups. Numerical experiments determined the best parameter combination settings to evaluate the algorithm performance. The conducted experiments contained a real dataset. The current study compared the proposed algorithm to several other classification schemes, including ABC classification, K-means, and PSO.

Much remains to explore for further research. First, hybridizing classification schemes with K-means may further improve efficiency. Moreover, incorporating classification schemes with other factors and criteria could provide more management flexibility. Finally, how to properly classify items at different stages of the supply chain and how it affects performance can be interesting subjects for further study.

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