Link Dynamic and Mobility Measure of a MANET with the Brownian Mobility Model

Yawgeng A. Chau and Yao-Hua Chen
Department of Communications Engineering, Yuan Ze University
Tao Yuan, 320 Taiwan
eeyaw@saturn.yzu.edu.tw and s948601@mail.yzu.edu.tw

ABSTRACT
To characterize the behavior of a mobile ad-hoc network (MANET), the Brownian mobility model is studied, where the random coordinators of mobile nodes are modeled by the Brownian motion. Based on the Brownian mobility model, the average relative speed of mobile nodes in a MANET cluster and the expected value as well as the variance of link lifetime are derived for node pairs. The expected link lifetime of node pairs is used to evaluate the average link lifetime of the MANET, and the mobility measure of the MANET is defined with the relative speed. Numerical and simulation results are presented for different cases.

KEY WORDS
MANET, Brownian Mobility, Link Lifetime, Relative Node Speed, Mobility Measure

1. Introduction

The MANET is becoming realizable, and has attracted research interests in various fields. The design of MANET protocols is usually based on specific mobility models such that the link behavior can be analyzed, e.g. the routing algorithm and protocol [1]. To characterize the motion behavior, different mobility and location models have been proposed. In [2] and [3], the authors have summarized the mobility models disclosed in the literature. In [4], the link dynamic for nodes moving in a constant speed has been analyzed, and different link statistics have been evaluated.
The link lifetime and the relative speed of mobile nodes in a MANET cluster play important roles in the protocol design for a MANET, where the relative speed can be employed to define the mobility measure of the MANET [5]. For example, if the mobility of the MANET is large and once a relay node of a multihop route moves out of the range from the source node, the link between the source and the relay will be broken, and a re-routing procedure needs to be started. In [6], the average lifetime of a link in a MANET was studied with deterministic, partially deterministic, and Brownian motion mobility models. Since the motion behavior of MANET nodes is essentially random, the model of Brownian motion can be employed to characterize the node mobility and thus the link behavior [6]-[9]. In fact, both the models of random walk [3] and random waypoint without pause time [1] are based on the Brownian motion. However, in [5], the Brownian motion is used to directly describe the range of two nodes, which results in the unrealistic infinite average link lifetime.
In this paper, the Brownian motion is used to model the random change of node coordinators in a MANET as depicted in Fig. 1 below, which will result in a satisfied prediction of the link lifetime as demonstrated by simulation results. In the context, the random coordinator of a MANET node is modeled by the standard Brownian motion. It can be proved that the distance of any pair of nodes in a MANET is a diffusion process [10], from which the expected value of the relative speed of mobile nodes as well as the expected value and the variance of the link lifetime for a node pair can be derived. Thus, the average link lifetime of the MANET and the corresponding mobility can be evaluated.

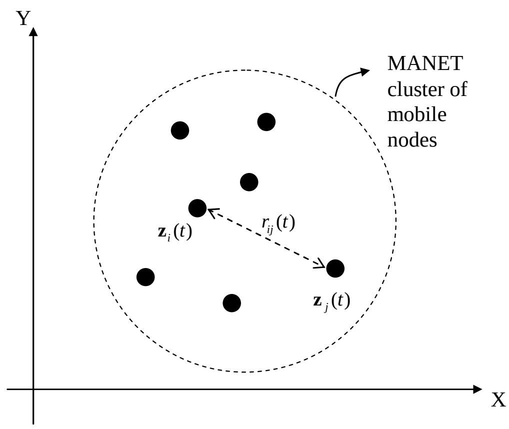

Fig. 1 The MANET cluster with random coordinators and relative speeds.

The paper is organized in the following way. In Section 2, the Brownian mobility model is presented for the characterization of the mobility behavior of a MANET. In Section 3, on the basis of the Brownian mobility model, the average link lifetime of a MANET as well as the expected relative speed of mobile nodes are derived, and the mobility measure is defined. In Section 4, the numerical and simulation results are presented for the average link lifetime and the mobility measure of different cases with various one-hop transmission ranges. Then, conclusions are drawn in Section 5.

2. Brownian Mobility Model

For a MANET that contains N nodes, let $\mathbf{z}_i(t)=[x_i(t),y_i(t)]$ be the random coordinator of node i at time t for $i=0,1,2,\cdots,N-1$ (see Fig. 1). Suppose the coordinator is characterized by the Brownian motion as

$$dx_i(t)=\sigma_i dw_i(t),\quad dy_i(t)=\sigma_i du_i(t);\quad i=0,1,2,\cdots,N-1 \quad (1)$$

where $w_i(t)$ and $u_i(t)$ are two independent Wiener processes, σ_i is the corresponding diffusion coefficient. In (1), $[w_i(t),u_i(t)]$ and $[w_j(t),u_j(t)]$ for $i\neq j$ are mutually independent. With the coordinator $(x_i(t_0),y_i(t_0))$ at initial time t_0, we have

$$x_i(t)-x_i(t_0)=\sigma_i w_i(t),\quad y_i(t)-y_i(t_0)=\sigma_i u_i(t) \quad (2)$$

The distance between $\mathbf{z}_i(t)$ and $\mathbf{z}_j(t)$ is given by

$$r_{ij}(t)=\sqrt{x_{ij}^2(t)+y_{ij}^2(t)} \quad (3)$$

where

$$x_{ij}(t)\triangleq x_i(t)-x_j(t),\quad y_{ij}(t)\triangleq y_i(t)-y_j(t) \quad (4)$$

In (4) and (5), $x_{ij}(t)$ and $y_{ij}(t)$ have independent Gaussian distributions individually with the identical variance $\sigma_{ij}^2 t=(\sigma_i^2+\sigma_j^2)t,$ and the means $x_i(t_0)-x_j(t_0)$ and $y_i(t_0)-y_j(t_0)$, respectively.

Let $x_{ij,0}=x_i(t_0)-x_j(t_0)$ and $y_{ij,0}=y_i(t_0)-y_j(t_0)$. By using the result of [10, p. 175] (also see [11, p. 491]), the probability density function (pdf) of $d_{ij,x}(t)$ conditioned on $r_{ij,0}=\sqrt{x_{ij,0}^2+y_{ij,0}^2}$ satisfies the Fokker-Planck equation

$$\frac{\sigma_{ij}^2}{2}\frac{\partial^2}{\partial r^2}f(r,t\mid r_0)-\frac{\sigma_{ij}^2}{2}\frac{\partial}{\partial r}\frac{f(r,t\mid r_0)}{r}=\frac{\partial}{\partial t}f(r,t\mid r_0) \quad (5)$$

where the Ito differential rule with the Levy oscillation property $(dw_{y,t})^2=(dw_{x,t})^2\doteq dt$ is used. The above differential equation has the following unique solution for the pdf $f(r,t\mid r_{0,ij})$

$$f(r,t\mid r_{0,ij}) =\frac{r}{(t-t_{0,ij})\sigma_{ij}^2}\exp\left(-\frac{r^2+r_{0,ij}^2}{2(t-t_{0,ij})\sigma_{ij}^2}\right)I_0\left(\frac{rr_{0,ij}}{(t-t_{0,ij})\sigma_{ij}^2}\right) \quad (6)$$

that is the Rician pdf with the parameters $r_{0,ij}$ and $(t-t_{0,ij})\sigma_{ij}^2$, where $I_0(\cdot)$ is the zero-order modified Bessel function of the first kind.

The distance given by (3) satisfies the following diffusion equation [11, p. 491, eqs. (6a)-(6c)]

$$dr_{ij}(t)=\frac{\sigma_{ij}}{2r_{ij}(t)}dt+\sigma_{ij}dw_r(t) \quad (7)$$

where w_r is another Wiener process.

With fixed $r_{0,ij}<R$ for $i,j=1,2,\cdots,N,$ when the distance $r_{ij}(t)$ is equal to or greater than the one-hop range R, it is claimed that the link between nodes i and j breaks, which may render another link generation later if $r_{ij}(t')<R$ for $t'>t.$

3. Analysis of Link Dynamic

3.1 Link Lifetime

For the link between nodes i and j, the time from the initial distance $r_{0,ij}$ to the first link breakage is defined as the link lifetime. Thus, the link lifetime is the random time required for $r(t)$ to reach R for the first time. For the node with the Brownian mobility, the link lifetime is also called the first passage time [13].

Denote the pdf of the first passage time by $p_{ij}(t).$ Let $F(\lambda,r)$ be the Lapalce transform of $f(r,t\mid r_{0,ij})$. Using [12, eq. (6.653.1)], we obtain

$$F(\lambda,r)=\int_0^\infty \frac{re^{-\lambda t}}{\sigma_{ij}^2 t}\exp\left(-\frac{r^2+r_{0,ij}^2}{2\sigma_{ij}^2 t}\right)I_0\left(\frac{rr_{0,ij}}{\sigma_{ij}^2 t}\right)dt =\begin{cases}\frac{2r}{\sigma_{ij}^2}I_0\left(\frac{\sqrt{2\lambda}r_{0,ij}}{\sigma_{ij}}\right)K_0\left(\frac{\sqrt{2\lambda}r}{\sigma_{ij}}\right) & r_{0,ij}\le r\\ \frac{2r}{\sigma_{ij}^2}K_0\left(\frac{\sqrt{2\lambda}r_{0,ij}}{\sigma_{ij}}\right)I_0\left(\frac{\sqrt{2\lambda}r}{\sigma_{ij}}\right) & r_{0,ij}\ge r\end{cases}. \quad (8)$$

where $K_0(\cdot)$ is the zero-order modified Bessel function of the second kind.

Then, applying [13, Theorem 3.1], we obtain the Laplace transform of $p_{ij}(t)$ as

$$P_R(\lambda)=\begin{cases}\frac{I_0\left(\sqrt{2\lambda}r_{0,ij}/\sigma_{ij}\right)}{I_0\left(\sqrt{2\lambda}R/\sigma_{ij}\right)} & r_{0,ij}\le R\\ \frac{K_0\left(\sqrt{2\lambda}r_{0,ij}/\sigma_{ij}\right)}{K_0\left(\sqrt{2\lambda}R/\sigma_{ij}\right)} & r_{0,ij}\ge R\end{cases} \quad (9)$$

In the following context, we consider the case where $r_{0,ij}<R$. Let T_L denote the corresponding link lifetime. Then, the average link lifetime is a function of $r_{0,ij}$, and can be evaluated as

$$\mu_L(r_{0,ij})=E[T_L]=-\frac{\partial P_R(\lambda)}{\partial\lambda}\Big|_{\lambda=0} =-\frac{\partial}{\partial\lambda}\frac{I_0\left(\sqrt{2\lambda}r_{0,ij}/\sigma_{ij}\right)}{I_0\left(\sqrt{2\lambda}R/\sigma_{ij}\right)}\Big|_{\lambda=0} =\frac{R^2-r_{0,ij}^2}{2\sigma_{ij}^2} \quad (10)$$

where, for any constant c, the relation

$$\frac{I_1\left(c\sqrt{x}\right)}{\sqrt{x}}\Big|_{x=0} = \left(cI_0\left(c\sqrt{x}\right) - \frac{I_1\left(c\sqrt{x}\right)}{\sqrt{x}}\right)\Big|_{x=0} \quad (11)$$

is used to obtain

$$\lim_{x\to 0}\frac{I_1\left(c\sqrt{x}\right)}{\sqrt{x}} = \frac{c}{2}\lim_{x\to 0} I_0\left(c\sqrt{x}\right) = \frac{c}{2} \quad (12)$$

The average link lifetime can also be derived using the differential equation given by [13, Theorem 6.1] with suitable boundary conditions.

Denoted by $\mu_L^{(2)}(r_{0,ij})$ the second moment of T_L that is also a function of $r_{0,ij}$ and satisfies the following differential equation [13, Theorem 6.1]

$$\frac{\sigma_{ij}^2}{2}\cdot\frac{d^2\mu_L^{(2)}(x)}{dx^2} + \frac{\sigma_{ij}^2}{2x}\cdot\frac{d\mu_L^{(2)}(x)}{dx} = \frac{r_{0,ij}^2 - R^2}{\sigma_{ij}^2}. \quad (13)$$

Solving (13) for $\mu_L^{(2)}(x)$, we obtain

$$\mu_L^{(2)}(r_{0,ij}) = \frac{1}{8\sigma_{ij}^4}(3R^4 - 4R^2 r_{0,ij}^2 + r_{0,ij}^4). \quad (14)$$

In consequence, from (10) and (14), the variance of T_L is

$$\sigma_L^2(r_{0,ij}) = \mu_L^{(2)}(r_{0,ij}) - \mu_2^2(r_{0,ij}) = \frac{R^4 - r_{0,ij}^4}{8\sigma_{ij}^4}. \quad (15)$$

The average link lifetime of the whole MANET with N nodes is defined by

$$\overline{T}_L = \frac{2}{N(N-1)}\sum_{i=1}^{N}\sum_{j=i+1}^{N}\mu_L(r_{0,ij}) = \frac{1}{N(N-1)}\sum_{i=1}^{N}\sum_{j=i+1}^{N}\frac{R^2 - r_{0,ij}^2}{\sigma_{ij}^2}. \quad (16)$$

A larger $\overline{T}_L$ implies that the nodes inside the MANET are well connected, and thus their relative mobility is low in some sense. Therefore, to measure the relative mobility for the MANET nodes, the relative mobility metric for these nodes may be defined with the relative speed of the mobile nodes.

3.2 Mobility Measure Based on the Relative Speed

Conditioned on fixed $r_{0,ij}$, the average relative speed between nodes i and j at time t is given by

$$\begin{aligned}\overline{\dot{r}_t} &= E[\dot{r}_t \mid r_{0,ij}] \\ &= \frac{1}{2t}\exp\left(-\frac{r_{0,ij}^2}{2\sigma_{ij}^2 t}\right)\int_0^\infty \exp\left(-\frac{r^2}{2\sigma_{ij}^2 t}\right) I_0\left(\frac{r r_{0,ij}}{\sigma_{ij}^2 t}\right) dr \quad (17)\\ &= \frac{\sqrt{\pi}\sigma}{2\sqrt{2t}}\exp\left(-\frac{r_{0,ij}^2}{4\sigma_{ij}^2 t}\right) I_0\left(\frac{r_{0,ij}^2}{4\sigma_{ij}^2 t}\right)\end{aligned}$$

where [12, eq. (6.618.4)] is used to obtain the integral.

For the time interval $[0,\tau]$, the average relative speed conditioned on fixed $r_{0,ij}$ is given by

$$\begin{aligned}\overline{\dot{r}(r_{0,ij})} &= \frac{1}{\tau}\int_0^\tau E[\dot{r}(t) \mid r(t), r_{0,ij}] f(r(t), t \mid r_{0,ij}) dt \\ &= \frac{\sqrt{\pi}\sigma_{ij}}{2\sqrt{2}\tau}\int_0^\tau \frac{1}{\sqrt{t}}\exp\left(-\frac{r_{0,ij}^2}{4\sigma_{ij}^2 t}\right) I_0\left(\frac{r_0^2}{4\sigma_{ij}^2 t}\right) dt \\ &= \frac{r_{0,ij}^3}{8\sigma_{ij}^2\tau^3} G_{23}^{21}\left(\frac{r_{0,ij}^2}{4\sigma_{ij}^2\tau}\left|\begin{matrix}-1,0\\ -1,-\frac{3}{2},-\frac{3}{2}\end{matrix}\right.\right) \quad (18)\end{aligned}$$

where [12, eq. (6.625.6)] is used to assess the integral and $G_{pq}^{mn}(\cdot)$ is the Meijer G-function [12]. Then, averaging (18) with respect to $r_{0,ij}$ yields the following average relative speed

$$\begin{aligned}\bar{\dot{r}} &= \frac{1}{R}\int_0^R \frac{r_{0,ij}^2}{8\sigma_{ij}^2\tau^3} G_{23}^{21}\left(\frac{r_{0,ij}^2}{4\sigma_{ij}^2\tau}\left|\begin{matrix}-1,0\\ -1,-\frac{3}{2},-\frac{3}{2}\end{matrix}\right.\right) dr_0 \\ &= \frac{R^3}{16\sigma_{ij}^2\tau^3} G_{34}^{22}\left(\frac{R^2}{4\sigma_{ij}^2\tau}\left|\begin{matrix}-1,-1,0\\ -1,-\frac{3}{2},-\frac{3}{2},-2\end{matrix}\right.\right) \quad (19)\end{aligned}$$

where [12, eq. (7.811.2)] is used to obtain the integral of G-function.

Consequently, the relative mobility metric for the MANET within the time interval $[0,\tau]$ can be defined by

$$\begin{aligned}\Lambda(R,\tau) &= \frac{2}{N(N-1)}\sum_{i=0}^{N-1}\sum_{j=i+1}^{N-1}\overline{\dot{r}_{ij}} \\ &= \frac{R^3}{8\tau^3 N(N-1)}\sum_{i=0}^{N-1}\sum_{j=i+1}^{N-1}\frac{1}{\sigma_{ij}^2} G_{34}^{22}\left(\frac{R^2}{4\sigma_{ij}^2\tau}\left|\begin{matrix}-1,-1,0\\ -1,-\frac{3}{2},-\frac{3}{2},-2\end{matrix}\right.\right) \quad (20)\end{aligned}$$

For the symmetric case where $\sigma_{ij}^2 = \sigma^2$ for all node i and j, the relative mobility metric given by (20) reduces to the form

$$\begin{aligned}\Lambda(R,\tau) &= \frac{2}{N(N-1)}\sum_{i=0}^{N-1}\sum_{j=i+1}^{N-1}\overline{\dot{r}_{ij}} \\ &= \frac{R^3}{16\sigma_{ij}^2\tau^3} G_{34}^{22}\left(\frac{R^2}{4\sigma_{ij}^2\tau}\left|\begin{matrix}-1,-1,0\\ -1,-\frac{3}{2},-\frac{3}{2},-2\end{matrix}\right.\right) \quad (21)\end{aligned}$$

4. Simulation and Numerical Results

In the numerical example, the symmetric case of $\sigma_i^2 = \sigma_j^2$ and $\sigma^2 = \sigma_{ij}^2 = 2\sigma_i^2$ for all $i, j = 1, 2, \cdots 20$ (i.e. $N = 20$) is considered, where the initial distances between all nodes are smaller than R. The initial coordinators of the MANET nodes are uniformly distributed on the circular area of diameter R in a random way. The analytic results evaluated with (16) are compared to the Monte-Carlo simulation results. The average link lifetime is plotted in Fig. 2 versus different values of σ^2, and in Fig. 3 versus different one-hop transmission ranges R. According to these results, a larger diffusion coefficient yields less link lifetime since the motion speed will be higher.

Then, the mobility measure of the symmetric case given by (21) for different values of the one-hop transmission range R is illustrated in Fig. 4, where $\tau = 1$ is used. From Fig. 4, the mobility increases very fast when the diffusion coefficient becomes large, particularly for smaller one-hop ranges.

5. Conclusion

In this paper, based on the Brownian mobility model, the link dynamic and the mobility measure of a MANET is studied, where closed-forms of the average link lifetime and the average relative speed of mobile nodes are derived. These link dynamic features can be applied to the design of MANET protocols. Although only the average link lifetime and the relative node speed are addressed in the paper, the Brownian motion can also be applied to assess other link behavior such as the link generation rate and the link change rate.

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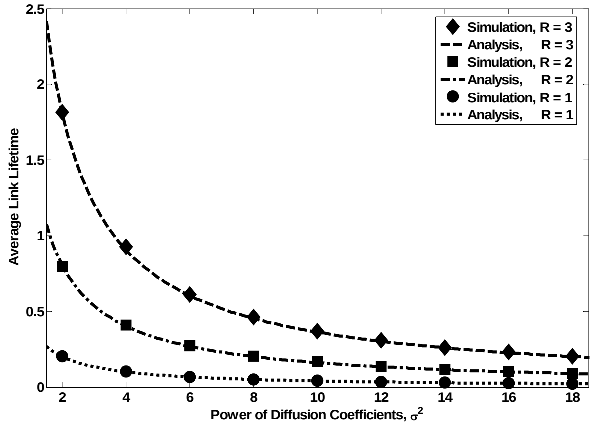

Fig. 2 The average link lifetime versus the diffusion coefficient.

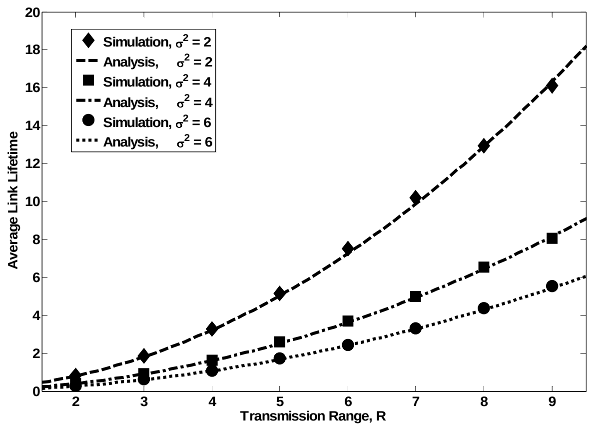

Fig. 3 The average link lifetime versus the one-hop range.

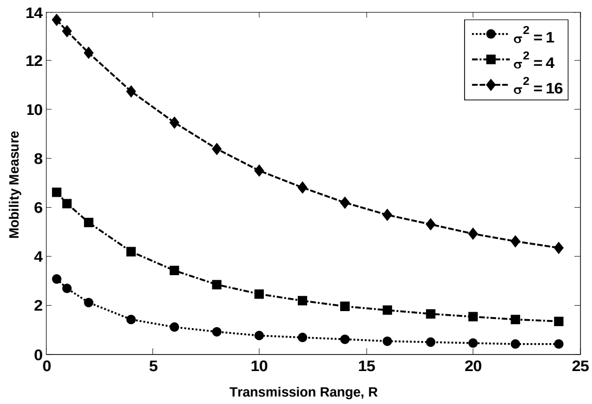

Fig. 4 The relative mobility measure for different one-hop ranges.