Anisotropic diffusion-based detail-preserving smoothing for image restoration

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Abstract

It is important in image restoration to remove noise while preserving meaningful detail such as blurred thin edges and low-contrast fine features. The existing edge-preserving smoothing methods may inevitably take fine detail as noise or vice versa. In this paper, we propose a new edge-preserving smoothing technique based on a modified anisotropic diffusion. The proposed method can simultaneously preserve edges and fine details while filtering out noise in the diffusion process. Since the fine details in the neighborhood of the image generally have larger gray-level variance than the noisy background, the proposed diffusion model incorporates both local gradient and gray-level variance to preserve edges and fine details while effectively removing noise. Experimental results from a variety of test samples including medical images and artwork images have shown that the proposed anisotropic diffusion scheme can effectively smooth noisy background, yet well preserve edge and fine details in the restored image.

Keywords: Image restoration; Anisotropic diffusion; Edge-preserving smoothing.

1. Introduction

Images are often suffered by noise in acquisition and transmission, which usually degrades the visual quality of the image. Many digital image processing applications require noise reduction and image restoration to produce more reliable results. To remove extraneous noise, denoising is often used as a pre-processing stage for segmentation, image analysis and interpretation. The difficulty of denoising is to avoid smoothing of edges, which are very important features of an observed image. Therefore, many edge-preserving smoothing methods have been proposed to reduce noise and restore the quality of an image for better visualization. The intention of edge-preserving smoothing is to effectively remove noise without blurring inter-region edges in the image. The edge-preserving techniques are widely used in various applications such as medical images, satellite images, and forensic images.

Many studies have been published on noise removal using edge-preserving methods. Median filtering is one of the most popular and easily-implemented methods because it performs well preserving edges while reducing noise, and is very computationally efficient. This method is applied to all pixels of the image with fixed filter coefficients, whether they are noisy or not. It may destroy fine details, and may produce streak and blotched effects in the restored image [1]. Hybrid median filter [2], also known as bidirectional linear median filter, gives data from different spatial directions with separate ranks. It can preserve edges much better than the classical median filter.

Other commonly used edge-preserving methods such as the Bayesian estimate, the Maximum Likelihood (ML) and Maximum A Posteriori (MAP) are used to estimate an original clean image from a corrupted image that contains Gaussian noise [3]. The main difficulties in MAP estimation are the choices of a proper prior distribution of the estimated image, and the corresponding energy function to be optimized. Markov random field (MRF) based methods [4] have also presented well denoising results without destroying edges. MRF methods maximize the posterior conditional probability distribution to restore an image. They require fairly accurate knowledge of the prior true image distribution. Most MRF-based methods are quite computationally expensive for parameter estimation.

Saint-Marc *et al.* [5] proposed an adaptive smoothing filter to remove noise while maintaining edge structures in an image. The core idea of adaptive smoothing is to use pixel intensities as the local attribute for discontinuity measuring. Tomasi and Manduchi [6] suggested restoring images by bilateral filtering that combined both domain and range filters. Domain filter considers the Euclidean distance between neighboring pixels and the central point. Range filter takes into account the intensity difference between pixels and the central point. When pixels have a large difference in intensity from the central point, they are weighted less even though they are very close to the central point in distance. Barash [7] pointed out that both adaptive smoothing and bilateral filtering can be regarded as special cases of anisotropic diffusion.

Anisotropic diffusion was first proposed by Perona and Malik [8] for scale-space description of images and edge detection. This approach is basically a modification of the linear diffusion, and the continuous anisotropic diffusion is given by

$$\frac{\partial I_t(x,y)}{\partial t} = \operatorname{div}[c_t \cdot \nabla I_t(x,y)] \qquad (1)$$

where $I_t(x,y)$ is the image at time *t*; div represents the divergence operator; $\nabla I_t(x,y)$ is the gradient of the image, and c_t represents the diffusion coefficient. The diffusion model in Eq. (1) will reduce to an isotropic diffusion equation if the diffusion coefficient c_t is a constant. It is then equivalent to convolving the image with a Gaussian smoothing filter. The idea of anisotropic diffusion is to adaptively choose c_t in different diffusion iterations so that intra-regions in an image become smooth while edges of inter-regions are preserved. The diffusion coefficient c_t is generally selected to be a nonnegative monotonically decreasing function of gradient magnitude. Small variations of intensity such as noise or shading with low gradient values can then be well smoothed, while edges with large intensity transitions are effectively retained.

The Perona-Malik anisotropic diffusion model cannot perform well for large noisy images simply based on point by point gradients. Catte *et al*. [9] proposed a modified anisotropic diffusion model with Gaussian smoothing operations. Their model estimates gradients window by window, and can effectively remove severely noisy points in the image. They also showed that the Perona-Malik anisotropic diffusion model is ill-posed, i.e., very similar images could produce divergent solutions. You *et al*. [10] gave an in-depth analysis of the behavior of the Perona-Malik anisotropic diffusion model by considering the anisotropic diffusion as the steepest descent method for solving an energy minimization problem. In order to get a more directional behavior of the diffusion process, Weickert [11] proposed an approach that replaces the scalar diffusion coefficient with a diffusion tensor. It steers the diffusion process according to the directional information contained in the image structure. This method can well preserve directional structure. Weickert [12] further proposed a revised diffusion model by analyzing coherence structures of edges. The coherence-enhancing diffusion model was also applied for medical image restoration [13]. The anisotropic diffusion approach has become a useful tool for edge detection [14, 15], image enhancement [16, 17], image smoothing [18, 19], image segmentation [20, 21], texture segmentation [22], defect detection [23] and image restoration [24, 25].

The classical anisotropic diffusion model of Perona and Malik in Eq. (1) only considers the local gradient information of each pixel in the image. That is, the Perona-Malik diffusion model smoothes the inner-region with lower gradient and stops the diffusion process at inter-region edges with higher gradient in the image. A high gradient magnitude is generally a good indication of edges, whereas a low gradient magnitude may not always point to non-edge regions or noise. Therefore, the gradient magnitude should not be used as the sole local feature in the diffusion process. The important local features defined in a small neighborhood of each pixel in the image may have low gradient such as blurred edges or fine details of an object, which should also be preserved during the diffusion process so that they will not mislead the postprocessing analysis and interpretation from the restored image. In order to retain fine details while removing noise, the local gray-level variance along with the gradient is added to a modified anisotropic model for image restoration. The proposed method can effectively filter out noise and preserve both edges and fine details of objects in the restored image.

The organization of this paper is as follows: Section 2 first overviews the Perona-Malik anisotropic diffusion equation. The proposed anisotropic diffusion scheme that adaptively smoothes or retains gray levels by taking into account both local gray-level variance and gradient for image restoration is then discussed. Section 3 presents experimental results from a variety of test samples including medical images, ancient artwork images and Lena image. This paper is concluded in Section 4.

2. Detail-preserving anisotropic diffusion

2.1 Perona-Mailk anisotropic diffusion model

Let $I_t(x,y)$ be the gray level at coordinates (x,y) of a digital image at iteration t, and $I_0(x,y)$ the original input image. The continuous anisotropic diffusion in Eq. (1) can be discretely implemented by using four nearest neighbors and the Laplacian operator [8]:

$$I_{t+1}(x,y) = I_t(x,y) + \frac{1}{4}\sum_{i=1}^{4}[c_t^i(x,y)\cdot \nabla I_t^i(x,y)] \qquad (2)$$

where $\nabla I_t^i(x,y)$, *i* = 1, 2, 3 and 4, represent the gradients of four neighbors in the north, south, east and west directions, respectively, i.e.,

$$\nabla I_t^1(x,y) = I_t(x,y-1) - I_t(x,y)$$

$$\nabla I_t^2(x,y) = I_t(x,y+1) - I_t(x,y)$$

$$\nabla I_t^3(x,y) = I_t(x+1,y) - I_t(x,y)$$

$$\nabla I_t^4(x,y) = I_t(x-1,y) - I_t(x,y)$$

$c_t^i(x,y)$ is the diffusion coefficient associated with $\nabla I_t^i(x,y)$, and is considered as a function of the gradient $\nabla I_t^i(x,y)$ in the Perona-Malik anisotropic diffusion model (P-M model), i.e.,

$$c_t^i(x,y) = g(\nabla I_t^i(x,y))$$

For the sake of simplicity, $\nabla I_t^i(x,y)$ is subsequently denoted by ∇I. The function $g(\nabla I)$ has to be a nonnegative monotonically decreasing function with $g(0)=1$ and $\lim_{|\nabla I|\to\infty} g(\nabla I) = 0$. The function $g(\nabla I)$ should result in low coefficient values at inter-region edges that have large gradient strength, and high coefficient values in non-edge regions that have low gradient strength. In the P-M anisotropic diffusion model, a possible diffusion coefficient function is given by

$$g(\nabla I) = 1 \big/ [1 + (|\nabla I| / K)^2] \tag{3}$$

where the parameter K is a constant, and must be fine-tuned for a particular application. Parameter K in the diffusion coefficient function acts as an edge strength threshold. If the K value is too large, the diffusion process will oversmooth and result in a blurred image. In contrast, if the K value is too small, the diffusion process will stop the smoothing in early iterations and yield a restored image similar to the original one. The classical P-M model considers only the gradient information of pixels for image restoration. It can thus preserve edges with large gradient strength, but inevitably smoothes both noise and fine details with low gradient strength in an ill-structured image. In the P-M model, the inter-region edges will be gradually smoothed as the diffusion iteration increases, even though the edges in the original image show very high gradient contrast. Therefore the traditional P-M model is very sensitive to the number of diffusion iterations. A careful selection of the number of iterations is required to ensure the success of the diffusion result. This is also a major drawback of the P-M model.

2.2. Proposed anisotropic diffusion model

In this paper, we propose a modified anisotropic diffusion model for image restoration. The proposed method not only effectively preserve edges, it can also well retain fine details while smoothing noise in an image. Figure 1(a) presents a shoulder patch image that contains a bear figure in the stuff (woven) background. This stuff material results in a non-uniform and noisy region in the sensed image. The objective of image restoration for such an image is to smooth the non-uniform background and preserve the shape and fine details of the bear. Figure 1(b) and 1(c) are two enlarged subimages from Figure 1(a). Figure 1(b) is a region from the noisy background. Figure 1(c) is a part of the bear, which contains the fine details of the fur. In Figure 1(a), the noisy background and the fur area (the embroidered detail) have similar gradient magnitudes, both with gradient values smaller than those of inter-region edges. The mean gradient magnitudes in Figure 1(b) and 1(c) are 24.31 and 24.88, respectively. The P-M diffusion model may smooth the fine embroidered detail when removing the noisy texture in the background. Although the gradient values in Figures 1(b) and 1(c) are similar, they show significant gray-level variations. The gray-level variations of Figures 1(b) and 1(c) are 350.81 and 1027.12, which are distinctly different compared with those of gradient magnitudes. The neighborhood in the inter-region edges generally has both high gradient magnitudes and high gray-level variances. The observation from this demonstrated image reveals that the fine detail and noisy background have smaller gradient strength with respect to the inter-region edges, and the variance in a fine detail area is generally larger than that in the noisy background. In order to preserve fine details while removing noise, both gray-level variance and gradient are considered as two local pixel features in the proposed diffusion model.

For a given pixel of coordinates (x,y) at iteration *t*, the gray-level variance is calculated from its 3×3 neighborhood, i.e.

$$\sigma_t^2(x, y) = \frac{1}{9}\sum_{i=-1}^{1}\sum_{j=-1}^{1}[I_t(x+i, y+j) - \bar{I}_t(x, y)]^2 \tag{4}$$

where $\bar{I}_t(x, y)$ is the mean of gray levels in the 3×3 neighborhood window. Since the range of gray-level variance is dramatically larger than that of gradient, the variance $\sigma_t^2(x,y)$ is first down-scaled so that both feature values of variance and gradient are compatible to each other. For an 8-bit gray-level image, the normalized variance is given by

$$\sigma_{t,N}^2(x, y) = 1 + \frac{\sigma_t^2(x, y) - \mathrm{Min}\sigma_t^2}{\mathrm{Max}\sigma_t^2 - \mathrm{Min}\sigma_t^2} \cdot 254 \qquad (5)$$

where $\mathrm{Min}\sigma_t^2$ and $\mathrm{Max}\sigma_t^2$ are the minimum and maximum gray-level variances of the diffused image at iteration t, respectively. By incorporating both local features of gray-level variance and gradient in the diffusion process, the diffusion coefficient function in Eq. (3) is revised as

$$g(\nabla I_t^i(x,y),\ \sigma_{t,N}^2(x,y)) = 1 \Big/ \left[1 + \left(\frac{\nabla I_t^i(x,y) \cdot \sigma_{t,N}^2(x,y)}{K_0}\right)^2\right] \qquad (6)$$

where K_0 is a positive constant used as an edge strength threshold. The edge strength threshold K_0 now completes with both the gradient and the additional gray-level variance in the revised diffusion coefficient function. The proposed diffusion model with the revised diffusion coefficient function of Eq. (6) is therefore given by

$$I_{t+1}(x,y) = I_t(x,y) + \frac{1}{4}\sum_{i=1}^{4}[g(\nabla I_t^i(x,y),\ \sigma_{t,N}^2(x,y)) \cdot \nabla I_t^i(x,y)] \qquad (7)$$

If $\sigma_{t,N}^2$ is fixed throughout the entire image, $\sigma_{t,N}^2 \big/ K_0$ will become a constant, and the modified diffusion model with the diffusion coefficient function defined in Eq. (6) will be equivalent to the P-M diffusion model.

Figure 2 demonstrates the effectiveness of the proposed diffusion model with respect to the traditional P-M diffusion model under a given number of diffusion iterations T=100 for the shoulder patch image shown in Figure 1(a). The result from the traditional P-M diffusion model (with K=8) is presented in Figure 2(a). We can observe that the background textures along with the fine details of the bear hair have been removed. The P-M model can effectively smooth the background and well preserve the contour shape of the bear. However, it loses the details within the region of the target object. The diffusion result from the proposed method (with K_0 =30) is presented in Figure 2(b). It shows that the background textures are effectively smoothed as a uniform region and both the shape and all the details of the bear are also well preserved by incorporating both local gray-level variance and gradient in the proposed diffusion model.

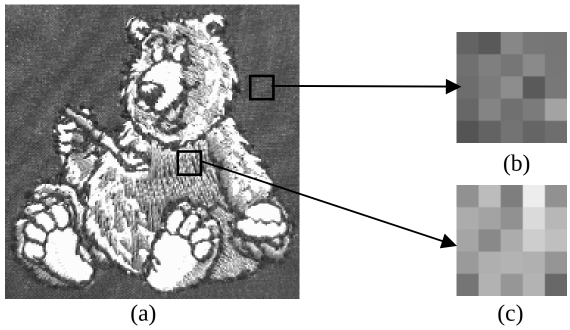


Fig. 1. A shoulder patch containing a bear figure: (a) original image; (b) sub-image from background area; (c) sub-image from fine hair area.

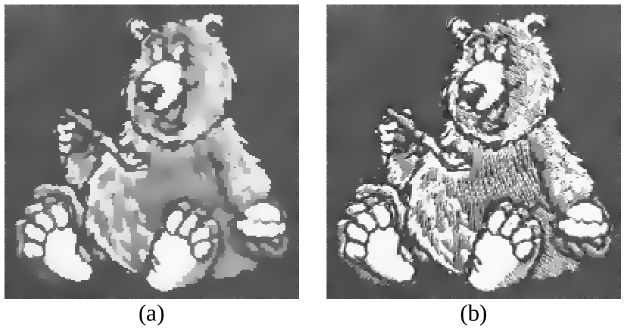


Fig. 2. Smoothing results of the shoulder patch shown in Figure 3(a): (a) diffusion result from the traditional P-M model (after 100 iterations with K=8); (b) diffusion result from the proposed model (after 100 iterations with K_0 =30).

3. Experimental results

In this section, we present experimental results from evaluating the efficacy of the proposed method. Test samples include medical image, Chinese painting and calligraphy images and the Lena image. The number of iterations is set to 100 for all test images in the following experiments. The diffusion result of each test image is presented by an appropriately selected parameter value K_0 (for the proposed method) or K (for the P-M model).

Figure 3(a) shows an instance of noisy computerized tomography (CT) image. Figure 3(b) presents the result from the P-M diffusion model. It well cleans noise but blurs the details of CT data in the restored image. It is not suitable for medical image analysis because the restoration ignores detailed features and may result in misdiagnosis. Figure 3(c) shows the result from the proposed diffusion model. It well preserves the details and inter-region edges while effectively removing noise in the CT image. The restoration result indicates that the proposed model can improve the visual quality of CT images.

(a) (b) (c)

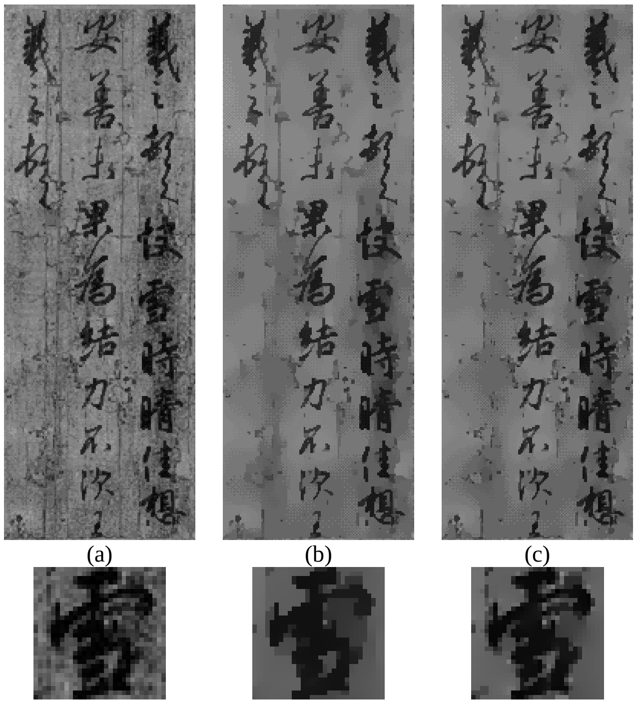
Fig. 3. Diffusion results of an CT image: (a) original CT image containing noise; (b) result of P-M diffusion model with K=2; (c) result of the proposed diffusion model with K_0 =3. (T=100.)

The proposed diffusion method can also be applied in image restoration for images of ancient artwork. Figure 4(a) depicts a Chinese painting from the Northern Sung Dynasty (960-1127 A.D.). In order to improve image quality, the mottled background should best be removed, yet the fine details well preserved. Figure 4(b) shows the result from the P-M diffusion model, in which the mottled background is effectively removed, but the feathers of the bird in the painting are also eliminated. Figure 4(c) presents the result from the proposed diffusion model. The diffusion result improves the mottled background, but also retains the detailed features in the painting.

(a) (b) (c)

Fig. 4. Diffusion results of a Chinese painting image: (a) Original image; (b) result of P-M diffusion model with K=4; (c) result of the proposed diffusion model with K_0 =12. (T=100.)

Figure 5(a) is a partial image of the Chinese calligraphy replicated in Tang Dynasty (618-907 A.D.). Figures 5(b) and (c) show, respectively, the results from the traditional P-M diffusion model and the proposed diffusion model. Figures 5(d)-(f) are enlarged sub-images from Figures 5(a)-(c). Each sub-image contains the Chinese character for "snow". It can be seen that some detailed strokes of the character are missing with the P-M model. The proposed method, however, can eliminate the speckled background but well preserve the shape of each character stroke.



(a) (b) (c)

(d) (e) (f)

Fig. 5. Diffusion results of a Chinese calligraphy image: (a) Original image; (b) result of the P-M diffusion model with K=4; (c) result of the proposed diffusion model with K_0 =25; (d)-(f) sub-images with Chinese character "snow" from (a)-(c), respectively. (T=100.)

In order to further verify the performance of the proposed method, various commonly used denoising and edge-preserving smoothing techniques including Gaussian smoothing, median filtering, bilateral filtering and the traditional P-M diffusion model are used for comparison. Figure 6(a) presents the Lena image containing random Gaussian white noise. Figure 6(b) shows the restoration result from the hybrid median filter, in which the edges cannot be effectively preserved and the noise is fused with the background. Figure 6(c) presents the restored image from the bilateral filter. It can be observed that most noise can be eliminated, but the contrast of the image is reduced and the inter-region edges are not sharply preserved. The restoration result in Figure 6(d) shows that the traditional P-M model has good diffusion effects in removing noise while preserving edges. However, the details of Lena's hair, eyes and lips are lost in the restored image. The coherence-enhancing diffusion can also remove noise while preserving edges, but it lost some fine and produced the flow-like pattern in the restored image, as seen in Figure 6(e). Figure 6(f) is the diffusion result of the proposed method. It clearly shows that the modified diffusion model can effectively maintain the subtle details and remove noise at the same time.

Fig. 6. Comparison of the restoration results from various filtering methods: (a) Lena image with noise; (b) hybrid median filter; (c) bilateral filter; (d) P-M diffusion model (K=3, T=100); (e) coherence-enhancing diffusion ($\sigma = 0.5, \rho = 6$, T=8); (f) proposed diffusion model (K_0 =15, T=100).

4. Conclusions

In this paper, we have proposed a modified anisotropic diffusion scheme for image restoration. It can effectively improve the quality of noisy images and also well preserve the inter-region edges and the fine details of the original image. The traditional P-M diffusion model only considers the gradient information of gray levels and, therefore, can not effectively preserve the fine details of the image. Since the region of subtle details generally involves large intensity variation in the image, the proposed diffusion method incorporates both the gray-level variance and gradient in the diffusion coefficient function. Experimental results have shown that the proposed anisotropic diffusion scheme can effectively remove noise, and yet maintain the sharp edges and fine details of a noisy image. The proposed method in its present form cannot be applied to the images that contain impulse noise. Such noisy pixels in the image generally involve higher gray-level variance and gradient than those of edges and fine details

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