Cumulative Count of Conforming Chart with Variable Sampling Intervals for Markov Dependent Production Processes

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Abstract

The cumulative count of conforming (CCC) chart is received amount of attention for automatically manufacturing processes recently. The CCC chart with variable sampling intervals (CCC_{VSI}) has shown to be more efficient than the conventional CCC chart with fixed sampling interval (CCC_{FSI}). In all of these studies, the production process is assumed to follow an independent and identically distributed Bernoulli pattern. However, it is common that the process characteristics become dependent in realistic situations, particularly when the production is automated.

In this research, a variable sampling interval procedure of the CCC chart is developed for a serially dependent process and the new scheme is more suitable for the modern manufacturing production processes.

Key words: CCC chart, Variable sampling interval, Markov dependent.

1. Introduction

Attribute control charts are usually used to monitor the fraction nonconforming of a process. However, the traditional $p$ chart has been shown to be inadequate in the monitoring of processes with low defect rates. Therefore, the cumulative counts of conforming chart (CCC chart) based on geometric distribution has been proposed as alternatives to the $p$ chart and has shown to be useful for processes monitoring in automated manufacturing.

The concept of CCC chart was first introduced by Calvin [2] and further developed by Goh and Xie [3,4,9,10,11]. Liu et al.[6] investigated the CCC chart with variable sampling intervals (CCC_{VSI}) and shown to have better performance than the conventional CCC chart with fixed sampling interval (CCC_{FSI}).

In current research of the CCC_{VSI} chart, the process is assumed to follow an independent and identically distributed Bernoulli pattern. However, the production of the conforming item may be serially dependent in realistic manufacturing. This paper investigates the CCC_{VSI} chart when the process is Markov dependent. The new scheme is more appropriate for practical production processes.
2. The CCC\textsubscript{VSI} chart for Markov dependent production processes

Before introducing the CCC\textsubscript{VSI} chart, the symbols and variables we used in this research are defined as follows:

\begin{itemize}
  \item \( p_0 \) the in-control process nonconforming rate
  \item \( p' \) the out-of-control process nonconforming rate
  \item \( \alpha \) the probability of false alarm
  \item \( X_i \) the cumulative count of conforming items inspected between the \((i-1)\)th and the \(i\)th nonconforming items
  \item \( n \) the number of different interval lengths of the CCC\textsubscript{VSI} chart
  \item \( d_j \) \( j = 1, 2, \ldots, n \) different sampling interval lengths of the CCC\textsubscript{VSI} chart
  \item \( D \) the sampling interval length of the CCC\textsubscript{FSI} chart
  \item \( \text{IL} \) the interval limits in the CCC\textsubscript{VSI} chart which divide the region between UCL and LCL into \( n \) sub-regions \( I_1, I_2, \ldots, I_n \)
  \item \( q_j \) the probability that point \( X_i \) falls in region \( I_j \) when the nonconforming rate is \( p_0 \).
  \item \( q'_j \) the probability that point \( X_i \) falls in region \( I_j \) when the nonconforming rate shifted to \( p' \).
  \item \( \text{ARL} \) the average run length
  \item \( \text{ATS}_{V} \) the in-control ATS of the CCC\textsubscript{VSI} chart
  \item \( \text{ATS}_{F} \) the in-control ATS of the CCC\textsubscript{FSI} chart
  \item \( \text{ATS}'_{V} \) the out-of-control ATS of the CCC\textsubscript{VSI} chart
  \item \( \text{ATS}'_{F} \) the out-of-control ATS of the CCC\textsubscript{FSI} chart
\end{itemize}

The performance of a control chart is usually evaluated by ARL. However, the time to signal is not

The upper control limit (UCL) and lower control limit (LCL) of the Markov dependent CCC chart can be calculated as follows:

\begin{align}
\text{UCL} &= \left[ 1 + \frac{\log(\alpha_u) - \log\left(\frac{1 - p_0}{1 - p_0(1 - d_1)}\right)}{\log\left[\frac{1 - p_0}{1 - p} \right]} \right] (1) \\
\text{LCL} &= \left[ 2 + \frac{\log\left(\frac{1 - \alpha_u}{1 - p_0}\right) - \log\left(\frac{1 - p_0}{1 - p_0(1 - d_1)}\right)}{\log\left[\frac{1 - p_0}{1 - p} \right]} \right] (2)
\end{align}

Where \( \alpha_u = P\{X \geq \text{UCL} \mid p = p_0\} \) and \( \alpha_l = P\{X \leq \text{LCL} \mid p = p_0\} \).

The VSI scheme divides the region between UCL and LCL into \( n \) sub-regions \( I_1, I_2, \ldots, I_n \) by \( n \) interval limits \( \text{IL}_{n-1} < \text{IL}_{n-2} \cdots < \text{IL}_2 < \text{IL}_1 \).

Supposing there are \( n \) different sampling interval lengths \( d_1, d_2, \ldots, d_n \) \( d_n < d_{n-1} < \cdots < d_2 < d_1 \) in the CCC\textsubscript{VSI} chart and let \( L_i \) be the sampling interval length used for inspection between the \((i-1)\)th non-conforming item and the \(i\)th non-conforming item. The value of \( L_i \) is dependent on the value of \( X_{i-1} \) and the relation is given as follows:

\begin{align}
L_i &= \left\{ \begin{array}{l}
  d_1, X_{i-1} \in I_1 = (\text{IL}_1, \text{UCL}) \\
  d_2, X_{i-1} \in I_2 = (\text{IL}_2, \text{IL}_1) \\
  \vdots \\
  d_n, X_{i-1} \in I_n = (\text{LCL}, \text{IL}_{n-1})
\end{array} \right. (3)
\end{align}

Where

\begin{align}
\text{IL}_1 &= \left[ 1 + \frac{\log(\alpha_u + q_1) - \log\left(\frac{1 - p_0}{1 - p_0(1 - d_1)}\right)}{\log\left[\frac{1 - p_0}{1 - p} \right]} \right] \\
\text{IL}_2 &= \left[ 1 + \frac{\log(\alpha_u + q_1 + q_2) - \log\left(\frac{1 - p_0}{1 - p_0(1 - d_1)}\right)}{\log\left[\frac{1 - p_0}{1 - p} \right]} \right] \\
\vdots \\
\text{IL}_{n-1} &= \left[ 1 + \frac{\log(\alpha_u + q_1 + \cdots + q_{n-1}) - \log\left(\frac{1 - p_0}{1 - p_0(1 - d_1)}\right)}{\log\left[\frac{1 - p_0}{1 - p} \right]} \right]
\end{align} (4)
a constant multiple of the number of samples to signal in VSI scheme. It is necessary to keep track of both the time and the number of samples inspected until a signal occurs. Average time to signal (ATS) and average number of items inspected (ANI) are two parameters used to evaluate the performance of the CCC\textsubscript{VSI} chart. Let $R$ be the number of points in the CCC\textsubscript{VSI} chart until an alarm arises. Then, the expected value of $R$ is the average number of points in the CCC\textsubscript{VSI} chart before an alarm arises including the point that gives the alarm. The ANI for both the CCC\textsubscript{VSI} chart and the CCC\textsubscript{FSI} chart can be calculated as:

$$\text{ANI} = E\left(\sum_{i=1}^{R} X_j\right) = E(R)E(X_j) = \frac{\text{ARL}}{p}$$

(5)

Therefore, $\text{ATS}_F$ can be calculated by using following equation with the sampling interval length $d$:

$$\text{ATS}_F = \text{ANI} \times d = \frac{\text{ARL}}{p} \times d$$

(6)

According to the $n$ variable sampling interval lengths of the CCC\textsubscript{VSI} chart, $\text{ATS}_V$ is considered as the expected value of the total time used before an alarm arises, which can be expresses as

$$\text{ATS}_V = E\left(\sum_{i=1}^{n} X_iL_i\right) = E(R)E(XL)$$

$$= \text{ARL} \left[ \sum_{j=1}^{n} E(X_j|L_i = d_j) P(L_i = d_j) \right]$$

(7)

When the process is in-control, it satisfies

$$P(L_i = d_j) = P\left\{X_{i-1} \in I_j \mid X_{i-1} \in (LCL, UCL)\right\}$$

$$= \frac{q_j}{1 - \alpha} = \frac{q_j}{q_1 + q_2 + \cdots + q_n}$$

(8)

for $i = 2, 3, \ldots, R$

The $\text{ATS}_V$ can be represented as

$$\text{ATS}_V = \frac{\text{ARL}}{p} \left[ \frac{d_1q_1 + d_2q_2 + \cdots + d_nq_n}{q_1 + q_2 + \cdots + q_n} \right]$$

(9)

3. Comparison of the CCC\textsubscript{FSI} and CCC\textsubscript{VSI} charts

The numerical results on the $\text{ATS}$ of both the CCC\textsubscript{VSI} and CCC\textsubscript{FSI} charts with $p_0 = 0.0001$ and $\alpha = 0.05$ are given in Table 1-2. As seen from Table1-2, the $\text{ATS}'_F$ and $\text{ATS}'_V$ increase when the serial correlation $d$ increases with the approximating values of $\text{ATS}'_F$ and $\text{ATS}'_V$. It means that the serial correlation has significant effect on the efficiency of detecting the process shift. Comparing the $\text{ATS}'_F$ with $\text{ATS}'_V$, $\text{ATS}'_V$ is constantly smaller than $\text{ATS}'_F$ under fixing the other parameters. That means the average time for the CCC\textsubscript{VSI} chart to detect the process shift is shorter than the CCC\textsubscript{FSI} chart. The detecting rate of the CCC\textsubscript{VSI} chart has a vast improved when the nonconforming rate grows up.

Table 1. $\text{ATS}_F$ with two sampling intervals

<table>
<thead>
<tr>
<th>$(d_1, d_2)$ = (1.9, 0.1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
</tr>
<tr>
<td>--------------</td>
</tr>
<tr>
<td>0.0001</td>
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<tr>
<td>0.0002</td>
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<tr>
<td>0.0003</td>
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<td>0.0004</td>
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<tr>
<td>0.0005</td>
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<tr>
<td>0.0010</td>
</tr>
<tr>
<td>0.0015</td>
</tr>
<tr>
<td>0.0020</td>
</tr>
</tbody>
</table>
Table 2. $\text{ATS}_V$ with two sampling intervals
$(d_1, d_2) = (1.9, 0.1)$

<table>
<thead>
<tr>
<th>$p$</th>
<th>$d=0$</th>
<th>$d=0.001$</th>
<th>$d=0.01$</th>
<th>$d=0.02$</th>
</tr>
</thead>
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<td>199693</td>
<td>199781</td>
<td>199727</td>
<td>199711</td>
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<tr>
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<tr>
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<tr>
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<tr>
<td>0.0015</td>
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<td>217</td>
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<td>713</td>
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<tr>
<td>0.0020</td>
<td>125</td>
<td>129</td>
<td>184</td>
<td>428</td>
</tr>
</tbody>
</table>

4. Conclusion

Considering both of the economic factors and practical condition in modern manufacturing, this paper proposes the variable sampling interval scheme to the conventional CCC chart, i.e. CCC$_{VSI}$ chart, when the process is Markov dependent. From evaluating the ATS of the CCC$_{FSI}$ chart and CCC$_{VSI}$ chart, the results of the CCC$_{VSI}$ chart reveals to have better performance in detecting the increase in the non-conforming rate $p$ than the CCC$_{FSI}$ chart.

References


